THE DESIGN OF A COMBINED STRESS FATIGUE JIG

ALLEN JOHNSTON GILMORE

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A. J. Gilmore

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by

Allen Johnston Gilmore
Lieutenant, United States Navy

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PREFACE

A search of the literature disclosed the limited amount of test data in the field of combined stress fatigue. This is probably due to the difficulties in production and operation of a suitable combined Stress Fatigue Machine. There are in existance several theories of failure under combined stresses, but not enough data for intelligent design criteria.

Between September 1952 and May 1953 the author designed and built a combined Stress Fatigue Jig to be used in conjunction with the Sonntag Universal Fatigue Testing Machine. This work was carried out at the United States Naval Postgraduate School, Monterey, California.

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TABLE OF SYMBOLS

B	Viscous damping coefficient
E	Young's modulus
G	Shear modulus
I, I_z	Moment of inertia
K, K_s, K_c	Spring constants
l	Length
M	Mass, Bending moment
M_t	Moment of torque
P	Force
P_o	Peak force
X	Displacement
X_o	Peak displacement
θ	Angle
χ	Angle
ω	Angular velocity
σ	Tensile stress
τ	Shear stress
δ	Deflection

LIST OF ILLUSTRATIONS

SUMMARY

The objective of this project was the design and production of a combined Stress Fatigue Machine for use at the U. S. Naval Postgraduate School. The design was carried out using the Sonntag Universal Fatigue Testing machine as the basic component and then producing a mechanism which would exert the desired stresses in a specimen.

Tests of the completed jig indicate that the assembled machine does function as predicted. The results of these tests are included as an appendix to this thesis. The errors indicated are attributable to the testing technique. It is believed that further calibration and testing will be able to reduce these errors greatly.

INTRODUCTION

In an effort to increase the efficiencies of mechanical apparatus, the trend has been continually to reduce the size and weight of machine parts, and to increase speeds. With the evolution of smaller and higher speed machinery, the phenomenon of fatigue has become an ever increasingly important limitation upon design. In the last few decades, much information has been gained by extensive research in the field of simple fatigue, and only recently has there been any research in the field of fatigue under conditions of multiple dynamic stresses. This is a practical trend as most machine parts are subject to more than one dynamic stress. In pursuit of this trend, the project of the design of equipment to run fatigue tests under multiple dynamic stresses was undertaken. Once having set upon this problem, and analyzing the methods of other people in the field, the project narrowed down to the design of a special jig for the Sonntag SF-1-U Universal Fatigue Testing Machine, which would produce simultaneous dynamic bending and torsion in a speciman.

FATIGUE TESTING IN GENERAL

There are available and in use throughout the world, many different fatigue testing machines which will give good results for simple fatigue. These consist of: bending machines which use rotating and non-rotating specimens loaded either in pure bending or as a cantilever, tension-compression machines, and torsion machines. Through the years much valuable data has been obtained. However, this data cannot be transposed to the field of combined stresses with any degree of adequacy for practical machine design.

PREVIOUS WORK IN FIELD OF COMBINED STRESS

Little data is available in the field of fatigue under combined stresses, due to the difficulty in producing and operating a combined fatigue machine. Gough (6) in England has done extensive work in this field using a resonant frequency machine acting on a specimen loaded as a cantilever.

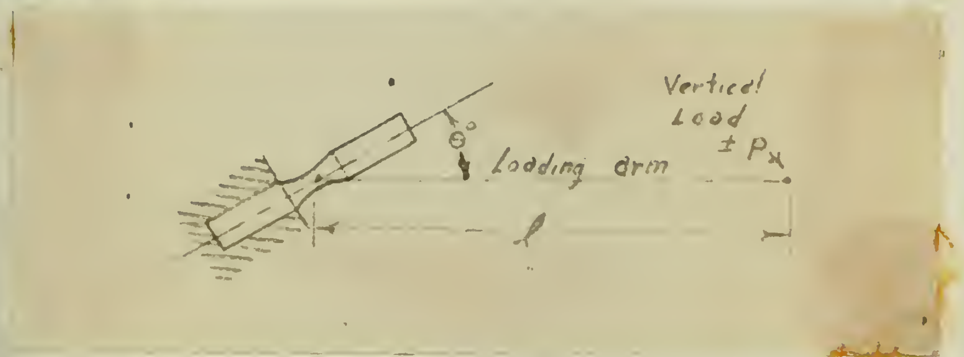

Figure 1

Schematic Diagram of Gough's Fatigue Machine

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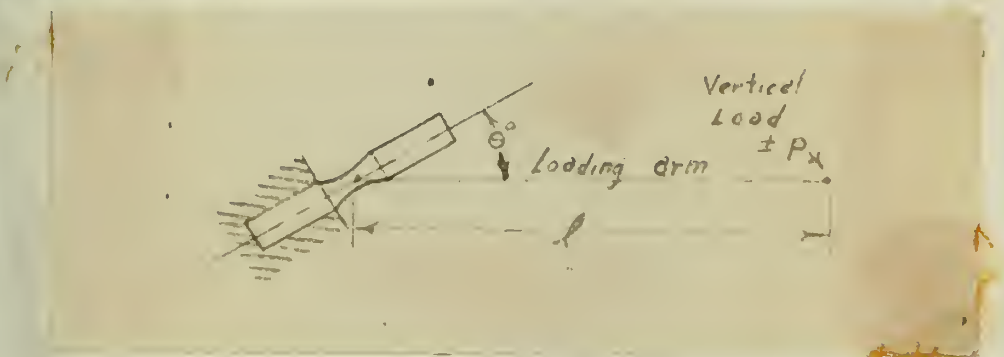

Figure 1

Schematic Diagram of Gough's Fatigue Machine

This machine uses a standard size specimen of non-constant cross section in the region under test, in order to predict with reasonable accuracy the stresses in the specimen at the point of failure. Sauer (14) did a limited amount of work by using the standard torsion jig of the Sonntag Universal Fatigue Testing Machine, and by removing the bearing adjacent to the torque arm. This machine again used a specimen of non-constant cross section loaded as a cantilever. In each

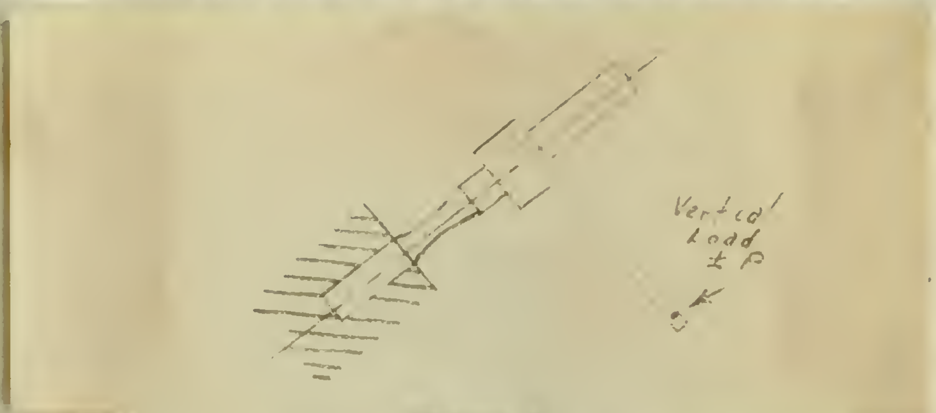

Figure 2

Schematic Diagram of Sauer's Fatigue Machine

case, the cantilever loading of the specimen necessitates a necked down section of the specimen in order to predict the point of failure and the stresses existing at that point. However, as the exact point of failure is a statistical function, not always occurring at the section of minimum cross section, exact determination of the stresses at the point of initial failure is impossible. Gough with a specially built machine and with a careful choice of specimen shape and size has produced excellent reproducible results, which are universally accepted. Sauer's results are of doubtful value because of the limited scope of test, and his failure to show adequately that the induced stresses were

as specified. In short, his jig had two degrees of freedom while the theory only considers one. It is perhaps significant that a large percentage of his specimens failed in the grips.

DESIRED FEATURES IN NEW TESTING MACHINE

In the design of a combined fatigue machine, there are many different conditions which must be satisfied. First it is desirable to use as many components "on hand" as possible from a standpoint of simplicity and economy. Secondly, it is desirable to have the specimen subjected to a dynamic force of constant amplitude rather than a constant amplitude of motion. This is desirable, because any change in physical properties or yield in a specimen of the constant amplitude of oscillation type specimen during a test will result in a test of unknown parameters. This leads directly to the desirability of using the Sonntag SF-1-U Universal Fatigue Testing Machine. This machine is capable of exerting a sinusoidal dynamic force varying in amplitude from zero to 1000 pounds, regardless of the rigidity of the test specimen. In addition this machine may be given a preset static force of from zero to 1000 pounds. Therefore the maximum setting would be sinusoidal force with peaks values of zero and 2000 pounds. In addition, the large table top of this machine with provision for fastening down various jigs provides a very flexible system for utilizing its full capabilities.

It is considered desirable to produce the tension in the specimen

by placing the test section in pure bending, to eliminate some of the undesirable features of previous machines which load the specimen as a cantilever. In addition the jig should be so designed that the stresses in specimen may be varied from pure tension (bending) to pure torsion so any desired ratio may be obtained. In view of the requirements the Sonntag Universal Fatigue Testing machine was chosen as the basic component of the combined fatigue testing machine.

THEORY OF SONNTAG UNIVERSAL FATIGUE TESTING MACHINE

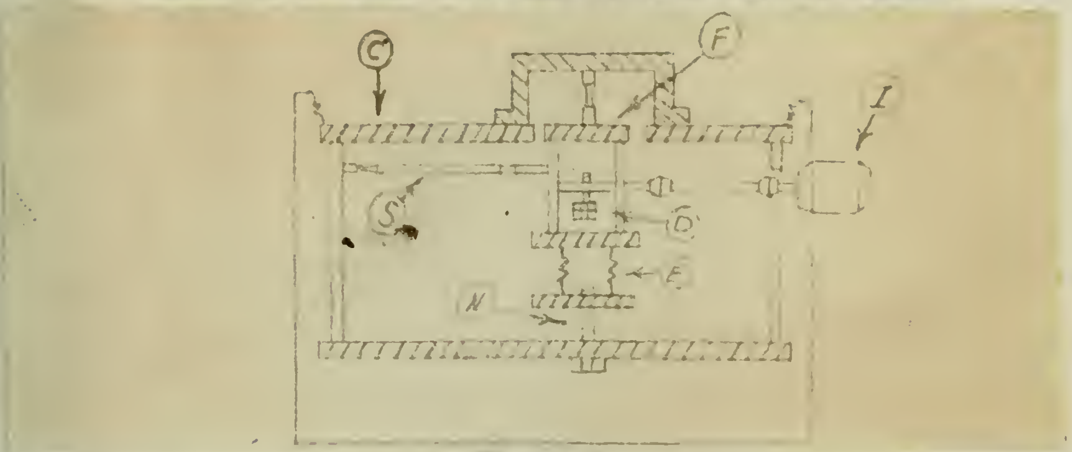

Figure 3

Sonntag Universal Fatigue Testing Machine

The function of the Sonntag Universal Fatigue Machine is to apply a vertical vibratory force to any specimen attached between the heavy stationary frame (C) and the reciprocating platen (F). This force can have any static component from zero to 1000 pounds either in tension or compression and any alternating component from zero to 1000 pounds. When operating in tension or compression alone, it is possible to have a maximum vibrating load fluctuating from zero to 2000 pounds.

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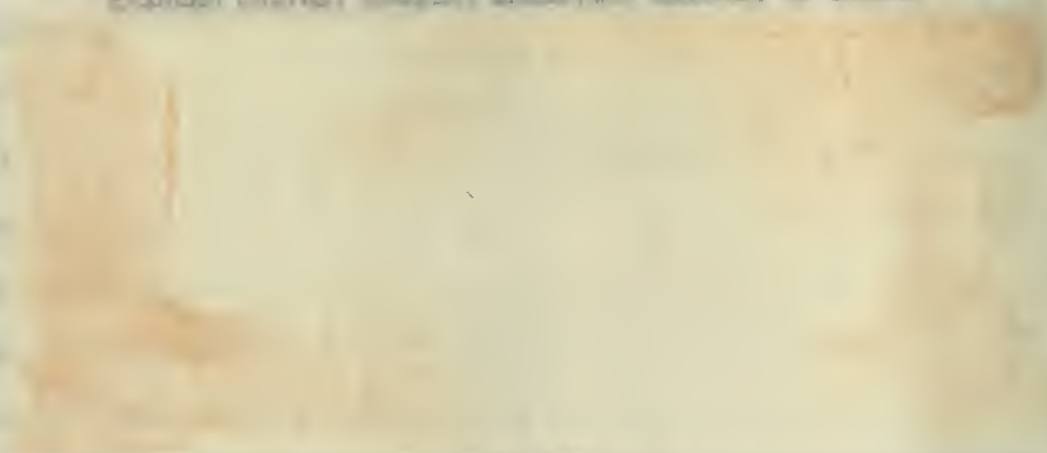

Figure 1

Sonntag Universal Fatigue Testing Machine

The function of the Sonntag Universal Fatigue Machine is to apply a vertical vibratory force to any specimen attached between the heavy stationary frame (S) and the reciprocating platen (P). This force can have any static component from zero to 1000 pounds either in tension or compression and any alternating component from zero to 1000 pounds. When operating in tension or compression alone, it is possible to have a maximum vibrating load fluctuating from zero to 2000 pounds.

The alternating force is produced by the unbalanced rotating mass (D) which is supported in the oscillating frame by two bearings. This mass is rotated through flexible couplings by an 1800 RPM synchronous motor (I). The amount of dynamic force exerted on the platen is set by adjusting the amount of eccentricity of the mass (D). Only the vertical component of this unbalanced force is exerted upon the specimen. The horizontal components are absorbed by guides with elastic hinges (S).

The Compensator springs (E) have one end attached to the oscillating frame and the other end attached to a plate which is in turn solidly fixed to the stationary frame. The purpose of these springs is to absorb all unknown inertia forces produced by the reciprocating masses and prevent these inertia forces from reaching the specimen. In order to function properly, the rigidity of the compensating springs is such that the natural frequency of the spring together with the reciprocating mass without specimen is the same as the operating frequency, 1800 cycles per minute. To accomplish this, provision is made for adjusting the mass to give resonance.

Preload is applied by adjusting the screw (N) which will place an initial tension or compression on the compensating spring. In addition there are micro switches to shut off the motor when the specimen fails and a counter to record the number of stress cycles to failure.

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The Compensator springs (K) have one end attached to the oscillating frame and the other end attached to a plate which is in turn solidly fixed to the stationary frame. The purpose of these springs is to absorb all unknown inertia forces produced by the reciprocating masses and prevent these inertia forces from reaching the specimen. In order to function properly, the rigidity of the compensating springs is such that the natural frequency of the spring together with the reciprocating mass without specimen is the same as the operating frequency, 1800 cycles per minute. To accomplish this, provision is made for adjusting the mass to give resonance.

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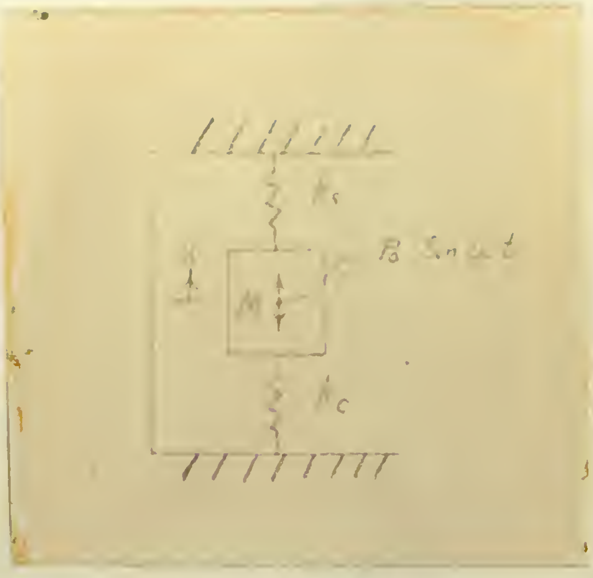

Figure 4

Schematic Drawing of Inertia Force Compensator

The schematic drawing of the testing machine as shown has the following equation of motion:

$$M \frac{\partial^2 x}{\partial t^2} + Kx = P_o \sin \omega t$$

where $K = K_s \; K_c$

K_s = spring constant of specimen

K_c = spring constant of compensating springs

X = displacement of Mass M

The steady state solution of this equation is:

$$x = x_0 \sin \omega t = \frac{P_o \sin \omega t}{K - M\omega^2}$$

Considering only peak values and substituting

$K_s + K_c$ for K we have:

$$X_o = \frac{P_o'}{(K_c + K_s) - M\omega^2}$$

however by design $K_c = M\omega^2$

Therefore $X_o = \dfrac{P_o}{K_s}$

Thus the force exerted on the specimen is that produced by the oscillating force irrespective of specimen rigidity of the specimen changes.

The mass of the reciprocating system was adjusted to give resonance without the specimen in place. The small additional rigidity of the specimen will raise the natural frequency of the system slightly. This is desirable to insure that the force and displacement are always in the proper phase.

The machine is driven by a synchronous motor to insure that the frequency of the forcing function is constant throughout the test. As the rigidity of the springs is fixed in manufacture some means of tuning the system must be provided. This is accomplished by adding tuning weights to the reciprocating frame such that the total mass, without specimen will place the system in resonance.

EFFECTS OF DAMPING

This analysis of the Universal Fatigue Testing Machine assumes that the vibrating system possesses zero damping. In his analysis of the machine B. J. Lazan (9) says, "In practically every case the damping in the system is sufficiently low to make the resulting errors

negligible". However the internal damping of all parts and particularly the inherent non-linear damping characteristics of anti-friction bearings cannot be so lightly ignored.

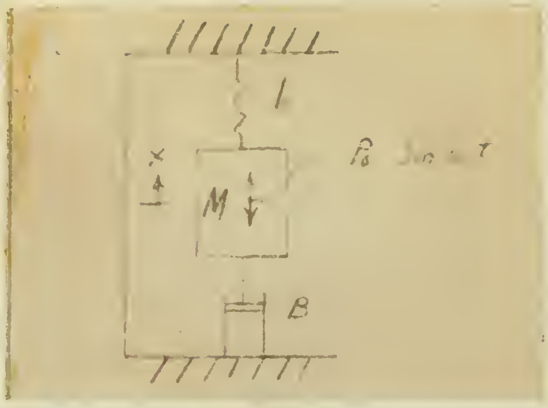

Figure 5

Schematic of System with Damping

If we consider all damping to be external viscous damping and analyze the system thoroughly, then the relative significance of this damping can be determined.

The equation of motion of this system is:

$$M \frac{d^2x}{dt^2} + B \frac{dx}{dt} + Kx = P_o \sin \omega t.$$

The steady state solution of this is:

$$x = \frac{P_o \sin(\omega t - \phi)}{\sqrt{(K - M\omega^2)^2 + \omega^2 B^2}}$$

where $\phi = \tan^{-1} \dfrac{B\omega}{K - M\omega^2}$

substituting $K_s + K_c$ for K and $K_c = M\omega^2$

$$x = \frac{P_o \sin(\omega t - \phi)}{\sqrt{K_s^2 + \omega^2 B^2}}$$

$$\phi = \tan^{-1} \frac{B\omega}{K_s}$$

From this it may be seen that the amplitude and hence the stresses in the specimen are no longer directly a function of the rigidity of the specimen and the forcing function. The significance of the error thus introduced into the system must be checked by some other means.

A check was made of the damping characteristic of the machine in three conditions: the machine alone, with the 15 1/4" torsion jig installed without a specimen, and the 15 1/4" torsion jig with an aluminum specimen. This was accomplished by placing an SR-4 strain gage on one of the elastic hinges of the machine and using the signal of this gage to operate a brush recorder. Then by causing the system to oscillate by compressing the compensating springs and releasing, the following damping coefficient were determined.

Machine alone	K= 0.05
Torsion jig without specimen	K= 0.125
Torsion jig with aluminum specimen	K= 0.130

where amplitude $X = X_0 e^{-Kt}$

To convert to the derived function B:

$$K = \frac{B}{2M\sqrt{\frac{Ks}{M} - \frac{B^2}{4M^2}}}$$

Assuming the spring constant $K_s \approx 10{,}000$ lb per in.

and the logarithmic decrement $K \approx 0.1$

then by symplifying $B^2 \approx \frac{25}{BKs}$

Then the expression $\sqrt{K_a^2+\omega^2 B^2}$ will approach K_s to within one part in 10^4 and can be considered equal to K_s with negligible error. Thus the damping in the standard Sonntage Fatigue Jig is considered negligible.

It is a reasonable assumption that the damping in the new jig will be of the same order of magnitude as in the existing machine and therefore be negligible. However, this will be thoroughly tested upon completion of the new jig.

The use of the Sonntag Universal Fatigue Testing Machine imposes several limitations. First the equivalent weight of the system must be within the limits of compensation of the machine (15.4 lbs.). Secondly the maximum amplitude of oscillation must not exceed 0.44 inches. Thirdly, the stress ranges desired in the specimen must be within the capability of the forces available.

DETAILS OF JIG DESIGN

The method chosen to accomplish the desired loading upon a specimen consists of mounting the specimen over the oscillating platen and supporting it with self aligning bearings which are attached to the platen. Attached to each end of the specimen is an arm which is capable of being swung in an arc to give any desired angle with the axis of the specimen. The ends of these arms are attached to the table top of the stationary frame.

A bending stress is imposed on the specimen by causing equal and opposite bending moments on the ends of the specimen. The moment is created by arms which have one end fastened to the fixed frame of the machine and one fixed to the oscillationg platen. By symmetry the forces are exerted equally on the two lever systems so that the specimen is placed in pure bending. A torsional stress is imposed through the same system. If the levers lie perpendicular to the axis of the specimen and are placed symmetrically, then as a vertical force is exerted on the platen this will cause a torsional force to be exerted upon the specimen. If the lever arms are placed in a positon between the axis of the specimen and perpendicular to the axis, then the combined stresses will be exerted on the specimen in a proportion which may be readily determined from the geometry of the system. These stresses will bear the same ratio for static as well as dynamic forces.

The bearing at the ends of the specimen must be capable of exerting a vertical force necessary to produce the required couples, but it also

must permit the bending and torsion to be fully transmitted to the specimen. For this reason self aligning bearings are used at this point. As the stresses in the specimen are changed and it deflects there is a small change in the axial length of the specimen. To eliminate any unknown axial forces, one of the vertical members holding these self aligning bearings must be made free to travel along the axis, while still exerting its vertical force. This is accomplished by hinging one of the bearing holders at the platen so that this second order motion can take place freely. Similarly deflection of the specimen will result in the motion of the extremity of the lever along its own axis in addition to the small motion along the axis of the specimen. As it is desirable to exert only a vertical force, self aligning bearings are placed at both ends of each holding down link.

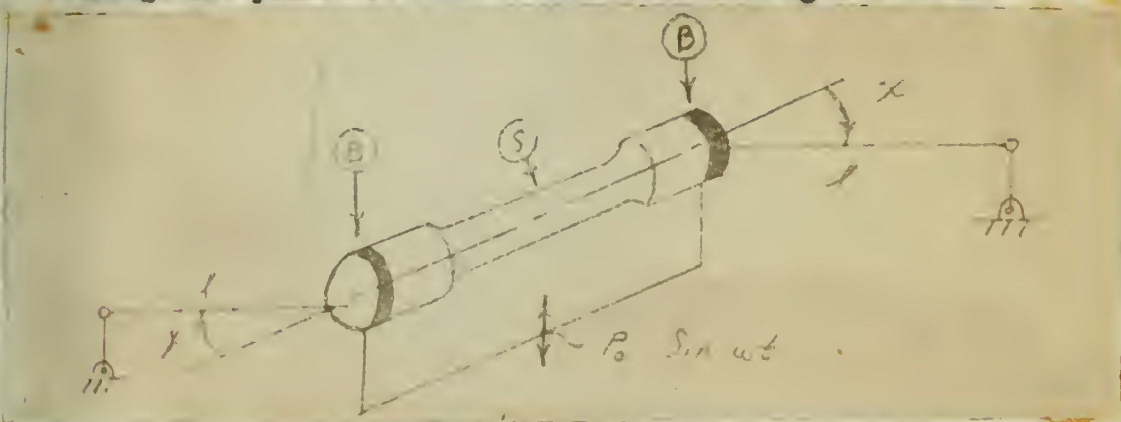

Figure 6

Schematic Drawing of Fatigue Jig Loading

The force P_o Sin wt is exerted on the specimen (S) through two self aligning bearings (B) mounted on either end of the specimen. This force is resisted by the two symetrically located torque arms.

By virtue of the geometry of this loading, the bending moment in the specimen will be $\frac{P_0}{2} \sin \omega t (l \cos \psi)$ and the moment of torque will be $\frac{P_0}{2} \sin \omega t. (l \sin \psi)$. Thereby varying the angle ψ the desired proportions of bending moment and torque, both dynamic and static, from pure bending to pure torsion may be exerted on the specimen.

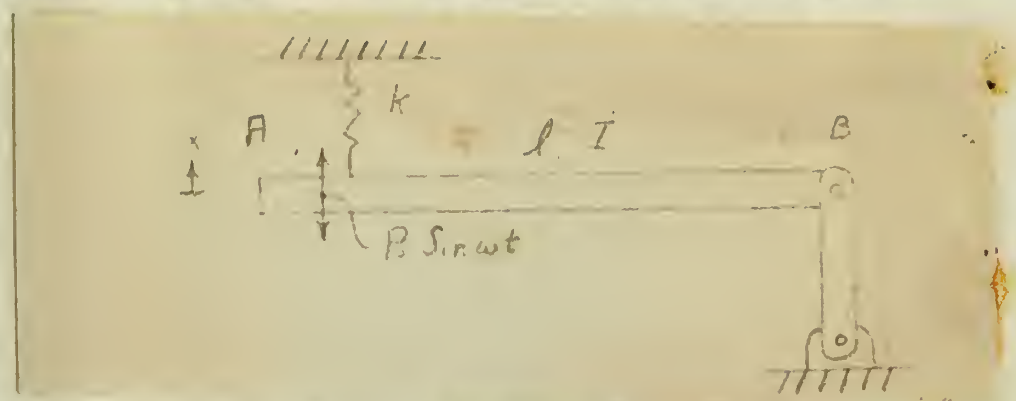

Figure 7

Schematic Drawing of Fatigue Jig

The configuration of the jig is such that the mass is not constrained so that the end A is forced to oscillate in a vertical path. Neglecting the small motion of the hinge B the equation of motion of this system is $\frac{I}{l^2} \frac{d^2 x}{dt^2} + Kx = P_0 \sin \omega t.$

The solution of this equation is:

$$X = X_0 \text{ Sin } wt = \frac{P_0 \sin \omega t}{K - \frac{I}{l^2} \omega^2}$$

if $\frac{I}{l^2} \omega^2 = Ke =$ a constant, which can be tuned for then

$$X = \frac{P_0 \sin \omega t.}{K_s}$$

or for peak deflections

$$P_o = K_s X_o$$

Therefore the inertia force compensator is still effective for this configuration. This system is not entirely accurate as there is a small axial motion of the hinges B. However, the error involved is negligible as will be shown later.

The specimen had to be checked so that the bending moments and torsional moments could be exerted upon it positively and with no lost motion. This was accomplished by having the moments exerted on the specimen holders. A tapered collar or collet fits over each end of the specimen and seats against a shoulder. The specimen is drawn into the specimen holders by a screw at each end which seats the tapered collar against a mating taper in the specimen holder. This allows the transmission of the moments to the specimen and eliminates the possibility of lost motion.

As shown in the description of the jig it is necessary to use the self aligning bearing in order to achieve the necessary degrees of freedom. The choice of these bearings was limited by the usual problems which are found in the use of antifriction bearing operating under reciprocating conditions. Any self aligning ball bearing when checked with the formula as recommended by Patterson (12) was either inadequate or too large. The only adequate bearing within the weight

limitations were aircraft type self aligning needle bearing. The choice of the bearing for the central bearing holder was simplified by the fact that similar bearings are used by Sonntag in their bending jig.

In order to meet the requirement that the jig be adjustable from pure bending to pure torsion, a false table top was necessary. This table top has circular slots to allow continuous adjustment.

Most oscillating components are manufactured from aluminum in an effort to remain within the weight limitations. However, parts such as the specimen holders and tapered collets are made of heat treated steel for strength.

The weak point in this jig is the strength of torque pins (Parts #8). Considering the force per inch of length (F) to be proportional to the distance from the center line of the specimen, and a case where the force exerted by the vertical link equal to five hundred pounds.

$$M = 2\int_{1}^{1.75} f\, x^2\, dx = 500 \times 6 \quad lb\ in$$

$$f = 515 \ \frac{lb}{in^2}$$

Therefore the moment at base of the cantilever pin is

$$M = \int_{0}^{0.75} 515(1+x)\, x\, dx = 214 \quad lb\ in.$$

then $\sigma = \pm \frac{Mc}{I} Kc = \pm \frac{214 \times 32}{\pi \left(\frac{3}{8}\right)^3} \times 1.5 = \pm 62{,}500\ PSI$

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Then $\sigma = \pm \frac{Mc}{I} K_c = \pm \frac{214 \times 32}{\pi \left(\frac{3}{8}\right)^3} \times 1.5 = \pm 62{,}500\ PSI$

However, this stress is computed for a cantilever bar with complete reversal of stress. In fact this section will not be so stressed because the shoulder of the pin will be set solidly against its seat, preventing this oscillating stresses from reaching this point of smallest cross section.

THEORETICAL CHECK OF SONNTAG REQUIREMENTS

It can be considered that the distortion is concentrated in the test specimen, and all other parts are of infinite rigidity. This is not true, but is sufficiently accurate to prove that the expected deflections are well within limitations of the Sonntage Universal Fatigue Testing Machine.

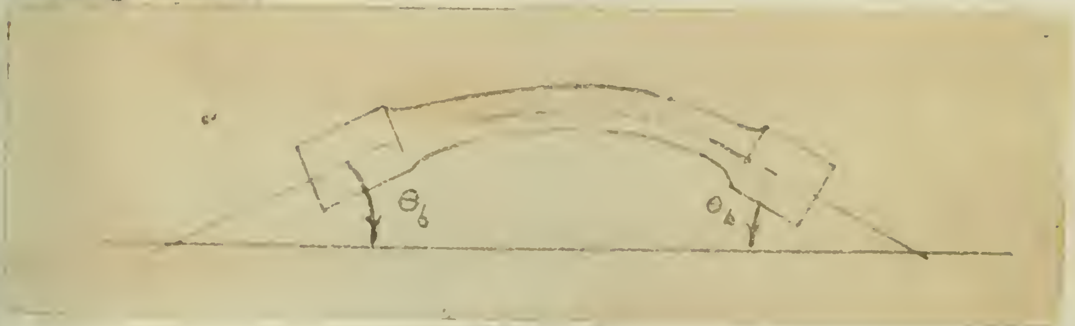

Figure 8

Distortion of Specimen

The distortion of the test section can be divided into two parts. One part due to bending, and one part due to torsion. In each case the center section can be soncidered as fixed due to the symmetry of the jig.

For the distortion due to bending

$$\theta_b = \frac{1}{2}\frac{M\ell}{EI_2} = \frac{1}{2}\frac{M\,\frac{13}{16}\times 64}{E\pi(\frac{1}{2})^4} + \frac{1}{2}\frac{M\,\frac{3}{8}\times 64}{E\pi(\frac{3}{4})^4} = 144.6\,\frac{M}{E}$$

Similarly the angle of twist from the center section is

$$\theta_t = \frac{1}{2}\frac{Mt\,32\ell}{G\pi d^4} = \frac{Mt}{G}\left[\frac{32\times\frac{13}{16}}{\pi(\frac{1}{2})^4} + \frac{32\times\frac{3}{8}}{\pi(\frac{3}{4})^4}\right. = 144.6\,\frac{Mt}{G}$$

The deflection experienced by the machine is relative vertical motion between the central self-aligning bearing and the hinge pins

at the extremity of the arms. For the case of pure bending of a steel specimen where it is desired to have an oscillating tensile stress of

100,000 PSI

$$M = \frac{\sigma I}{c} = \frac{100{,}000\ \pi \left(\frac{1}{2}\right)^3}{32} = 1{,}230 \text{ lb. in.}$$

The length of the arm is 7.668 in.

so force of platen P_o = $2X \dfrac{1{,}230}{7.668}$ = 321 lb.

and the deflection of platen

$$\delta_s = \pm \left[144.6 \times \frac{1230}{30^6} \right] 7.668 = \pm\ 0.0455 \text{ in.}$$

Similarly if aluminum is subjected to the same loading

$$\delta_{al} = \pm\ 0.124 \text{ in.}$$

If it is desired to stress the specimen simultaneously with 100,000 PSI tensile stress at $\pm$ 50,000 PSI shear stress the following will be the case

$$\tau = \frac{16 M_t}{\pi d^3}$$

$$M_t = \frac{50{,}000\ \pi \left(\frac{1}{2}\right)^3}{16} = 1{,}230 \text{ lb in.}$$

First it is necessary to find the arm angle and force P.

$M = (1.668 \quad 6 \cos \chi)\ P = 1230$ lb. in.

$M_t = 6 (\sin \chi)\ P = 1230$ lb. in.

$\chi = 56.5^{\circ}$

$P = 246$ lb.

Therefore the machine must be set for $\gamma = 56.5°$

And force $P_o = \pm\ 492$ lb.

$$\therefore\ \delta_s = \pm \left\{\left[144.6 \times \frac{1230}{3016} \times \frac{1230}{246}\right] + \left[144.6 \times \frac{1230}{1156} \times \frac{1230}{246}\right]\right\}$$

$$\delta_s = \pm\ 0.1104 \text{ in.}$$

similarly for aluminum

$$\delta_{al} = \pm\ 0.30 \text{ in.}$$

If it is desired to subject a steel specimen to a shear stress of 50,000 PSI, again P and γ must be determined

$M = (1.668 + 6 \cos \gamma)\ P = 0$; $M_t = 6(\text{Sin } \gamma)\ P = 1230$ lb. in.

$\gamma = 106.3°$ $\qquad P = 214$ lb.

so setting of machine will be $\gamma = 106.3°$

and force $P_o = 428$ lb.

All of these computations assume no internal damping in the machine or in the specimen. As stated before this will be investigated for accuracy at a later date.

To further justify neglecting the motion of the outer hinge first consider the inertia force imparted by the arm and link on the specimen. By considering the mass of the links concentrated at the ends of links as outlined by Den Hartog (2) in "Mechanical Vibrations" and finding the accelerations of the ends neglecting the residual inertia torque, the inertia forces can be computed. As the axial inertia force is greatest in the pure bending configuration, this will be investigated.

The concentrated masses will be:

Figure 9

Schematic of Jig

for a vertical displacement of platen of 0.3 Sin $60\pi t$ in., the horizontal displacement of the bearing will be $l[1-\cos\theta]$

where

$$\theta = \frac{0.3}{7.668}\sin 60\pi t = 0.039 \sin 60\pi t \text{ radians}$$

expanding into a series

$$l[1-\cos\theta] = l\left[\frac{\theta^2}{2!} - \frac{\theta^4}{4!} + \frac{\theta^6}{6!} - \frac{\theta^8}{8!} \cdots\right]$$

$$= 7.668\left[\frac{0.0015 \sin^2 60\pi t}{2} - \frac{0.0000023 \sin^4 6\pi t}{24} \cdots\right]$$

considering only the first term as significent the equations of the m motion of the pivot are:

$$y = 0.0058 \sin^2 60\pi t = 0.0058 \sin^2 188t$$

$$\dot{y} = 2.18 \sin 188t \cos 188t$$

$$\ddot{y} = 400 \cos^2 188t - 400 \sin^2 188t = 400 \cos 376t$$

The axial inertia force exerted upon the specimen will then be

$$F_i = -\frac{2.76}{386} \times 400 \cos 376t$$

$$= -2.95 \cos 376 \quad \text{lb.}$$

Thus the specimen has a uniform compressive stress of 15 PSI the extremities of the stress stroke. In general the amplitude oscillation will be much less and the resultant inertia forces also be much less. However even the 15 PSI is within the limit of error of the fatigue testing in the range of stresses necessary to produce this deflection ($\pm$ 240,000 PSI for aluminum).

The inertia forces of the jig which are reflected back into the inertia force compensator must be within the limit of tuning of the machine, as the machine is presently able to accommodate any jig with an equivalent weight of 15.4 pounds. To check the expected equivalent weight of this jig in the pure bending postion, the masses of the various links are considered concentrated at the pivot points as before. All parts of the jig which are rigidly attached to the platen will oscillate with it and so will have an equivalent mass equal to their mass. The equivalent mass then for the pure bending condition is

Aluminum in center	3.75
2 bearings (CFI)	0.80
Specimen	0.50
Weight of arm and specimen holder concentrated at bearing	6.65
	11.70 lbs.

As the machine is tuned for an equivalent weight of 15.4 pounds then 3.7 pounds of added weight is necessary in this condition to tune

the machine to the synchronous frequency. The equivalent weight of this system will vary slightly from pure bending to pure torsion but will be well within the limits of compensation of the machine.

ACTUAL CHECK OF OPERATING JIG

Upon completion of the jig it was assembled and operated with a specimen of negligible weight and rigidity to determine the equivalent weight of the jig in various configurations. This specimen consisted of a 1/8" diameter steel rod with a brass adapter on each end to attach to the specimen holding bolts. Having the curve of equivalent weights, a steel specimen with SR-4 type A-7 strain gages, was installed and tested. These strain gages were located such that two were located along the axes of maximum bending stress, and two along the axes of maximum strain from torsion and along the zero bending strain axes. With this configuration the two bending gages were not effected by torsion and the two sheer gages were not effected by bending. These gages were connected to two channels of a Hathway oscillograph for dynamic strain recording. The other half of the bridge circuits consisted of Baldwin SR-4 Strain Calibration Units which facilitated the amplitude calibration for each run.

The machine was then operated in various ratios of bending and torsion and the results computed (see Appendix). The errors found can be attributed to the technique of testing. A check of the stresses found and the predicted stress would indicate that the torque arms were not positioned exactly. For example, consider the readings for $\chi = 60^{\circ}$. If it is assumed that the actual arm angle was 59°, the predicted bending stress and sheer stress now being $\sigma_b = 19{,}350$ P.S.I. and $\tau = 10{,}400$ P.S.I., then the error involved would be $+1.8\%$ for bending and $+0.95\%$ for torsion. This will indicate that the arms were not set exactly

ACTUAL CHECK OF OPERATING JIG

Upon completion of the jig it was assembled and operated with a specimen of negligible weight and rigidity to determine the equivalent weight of the jig in various configurations. This specimen consisted of a 1/8" diameter steel rod with a brass adapter on each end to attach to the specimen holding bolts. Having the curve of equivalent weights, a steel specimen with SR-4 type A-7 strain gages, was installed and tested. These strain gages were located such that two were located along the axes of maximum bending stress, and two along the axes of maximum strain from torsion and along the zero bending strain axes. With this configuration the two bending gages were not affected by torsion and the two shear gages were not affected by bending. These gages were connected to two channels of a Sanborn oscillograph for dynamic strain recording. The other half of the bridge circuits consisted of Baldwin SR-4 Strain Calibration Units which facilitated the amplitude calibration for each run.

The machine was then operated in various ratios of bending and torsion and the results computed (see Appendix). The errors found can be attributed to the techniques of testing. A check of the stresses found and the predicted stress would indicate that the torque arms were not positioned exactly. For example, consider the readings for $\lambda = 60°$. If it is assumed that the actual arm angle was 30°, the predicted bending stress and shear stress now being $\sigma_b = 17,250$ P.S.I. and $\tau = 10,400$ P.S.I., then the error involved would be +1.5% for bending and +0.95% for torsion. This will indicate that the arms were not set exactly

throughout the test and the actual error involved is much less than indicated. Further testing and more accurate positioning of these arms will undoubtedly reduce these errors considerably.

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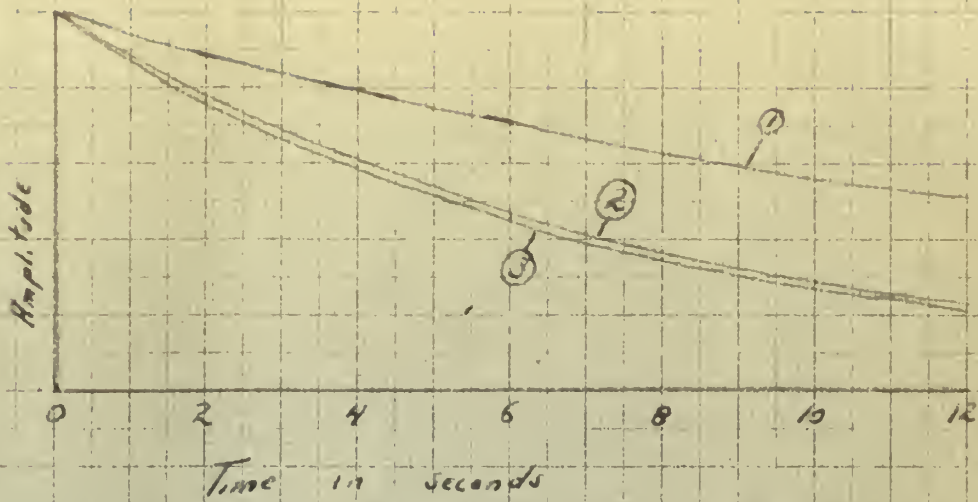
APPENDIX

TRANSIENTS

One of the greatest drawbacks to the inertia force type of Fatigue Testing machines is the presence of transients while bringing the testing machine up to synchronous speed. In general by the time that the transients have died out the test specimen has an unknown history or work hardening.

Perhaps, one method of eleminating or reducing this phenomenon would be the addition of a magnetic damping circuit. This would consist of a strong magnetic field through the platen while the motor is being brought up to speed. The magnetic field could then be broken at a time that the force is passing through neutral, or reduced gradually depending upon the best procedure as determined by test. A simple electro-magnet could easily be installed inside the cabinet to do just this function.



Envelope of
Damping curves of Sonntag Machine

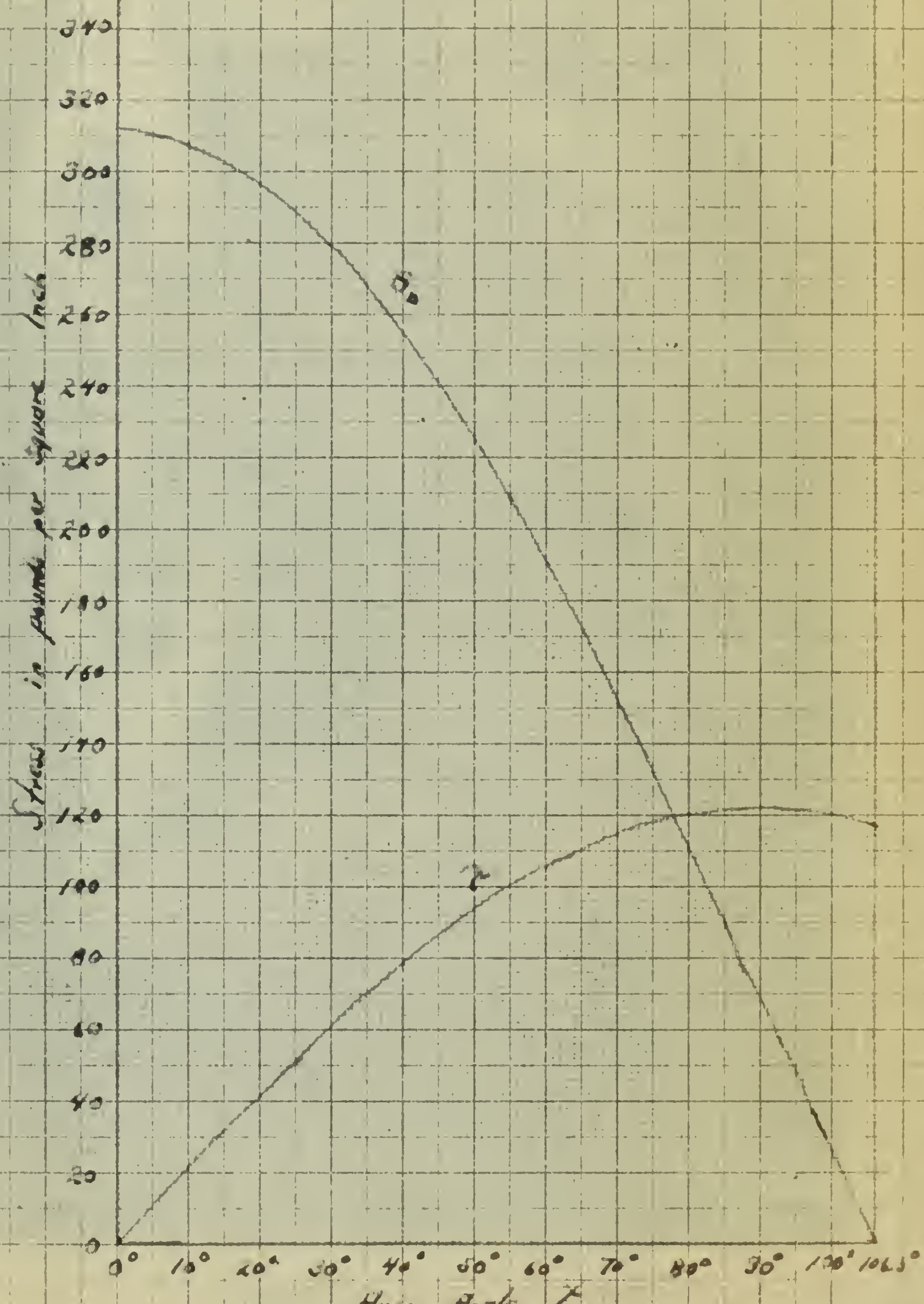
① Machine alone $K = 0.057$

② Machine with standard Torsion Jig without specimen $K = 0.125$

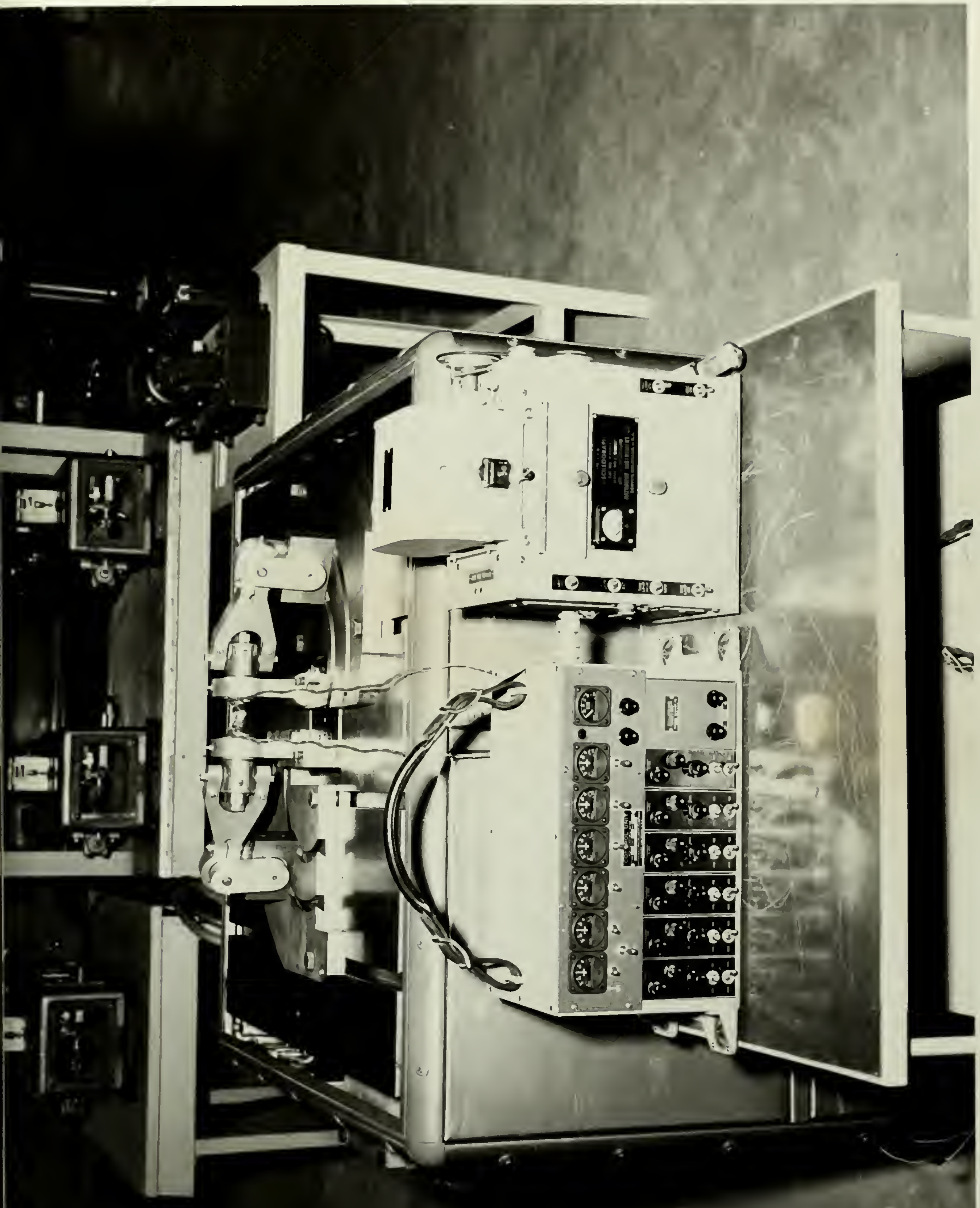
③ Machine with standard Torsion Jig with Aluminum specimen $K = 0.130$

Where $A = A_0 e^{-Kt}$

Maximum Tensile Stress due to Bending, σ_b
and Shear Stress due to Torsion, τ
Vs.
Arm angle ψ
per Pound of Static or Dynamic Force
For a 0.500 inch dia Specimen



ASSEMBLED JIG



ASSEMBLED JIG SHOWING INSTRUMENTATION

χ	l_{60}	l_{70}	M	M_t	σ_1	τ
0°	7.6855	0	3.8427	0	312.0	0
10°	7.5355	1.042	3.7977	0.5210	308.0	21.2
20°	7.3255	2.050	3.6627	1.0250	297.5	41.8
30°	6.8805	3.000	3.4403	1.5000	279.5	61.0
40°	6.2805	3.855	3.1403	1.9275	255.0	78.5
50°	5.5405	4.955	2.7703	2.4775	225.0	93.5
60°	4.6855	5.195	2.3427	2.5975	190.1	105.6
70°	3.7355	5.640	1.8677	2.8200	151.7	114.8
80°	2.7275	5.910	1.3637	2.9500	111.0	120.2
90°	1.6855	6.000	0.8427	3.000	68.4	122.0
100°	0.6435	5.910	0.3217	2.9500	26.2	120.2
106.3°	0	5.755	0	2.8775	0	117.0

EQUIVALENT WEIGHT

χ	Comp. Wt.	Equ. Wt.
0°	4.4	11.0
15°	4.6	10.6
30°	5.2	10.2
45°	5.6	9.8
60°	5.8	9.6
90°	6.0	9.4
106.3°	6.1	9.3

STRESSES MEASURED ON SPECIMEN

χ	Calib. Bending	Calib. Torsion	Amp Bending	Amp Torsion	Bending	Torsion	Bending	Torsion
0°			2.71		1.034		1.06	
30°(1)	0.800	0.945	3.075	0.98	0.96	0.259	0.985	0.265
30°(2)	0.76	1.00	2.88	1.02	0.948	0.255	0.973	0.262
60°	1.10	1.30	2.86	2.34	0.650	0.451	0.667	0.463
90°	1.06	1.36	0.95	2.80	0.224	0.515	0.230	0.529
106.3°	1.05	1.40	0.15	2.79	0.0357	0.497	0.0366	0.510

Assuming E = 29.5 lb/in^2 μ = 0.3

			Predicted		% error	
χ	σ_0	τ	σ_0	τ	σ_0	τ
0°	31,300	0	31,200	0	+ 0.32	–
30°(1)	29,100	6,010	27,950	6,100	+ 4.1	- 1.5
30°(2)	28,700	5,950	27,950	6,100	+ 2.6	- 0.25
60°	19,700	10,500	19,010	10,560	+ 3.5	- 0.57
90°	6,800	12,000	6,840	12,200	- 0.58	- 1.6
106.3°	1,080	11,600	0	11,700	(+)	- 0.94

Data obtained using

Baldwin SR-4 Strain Gages Type AR-7

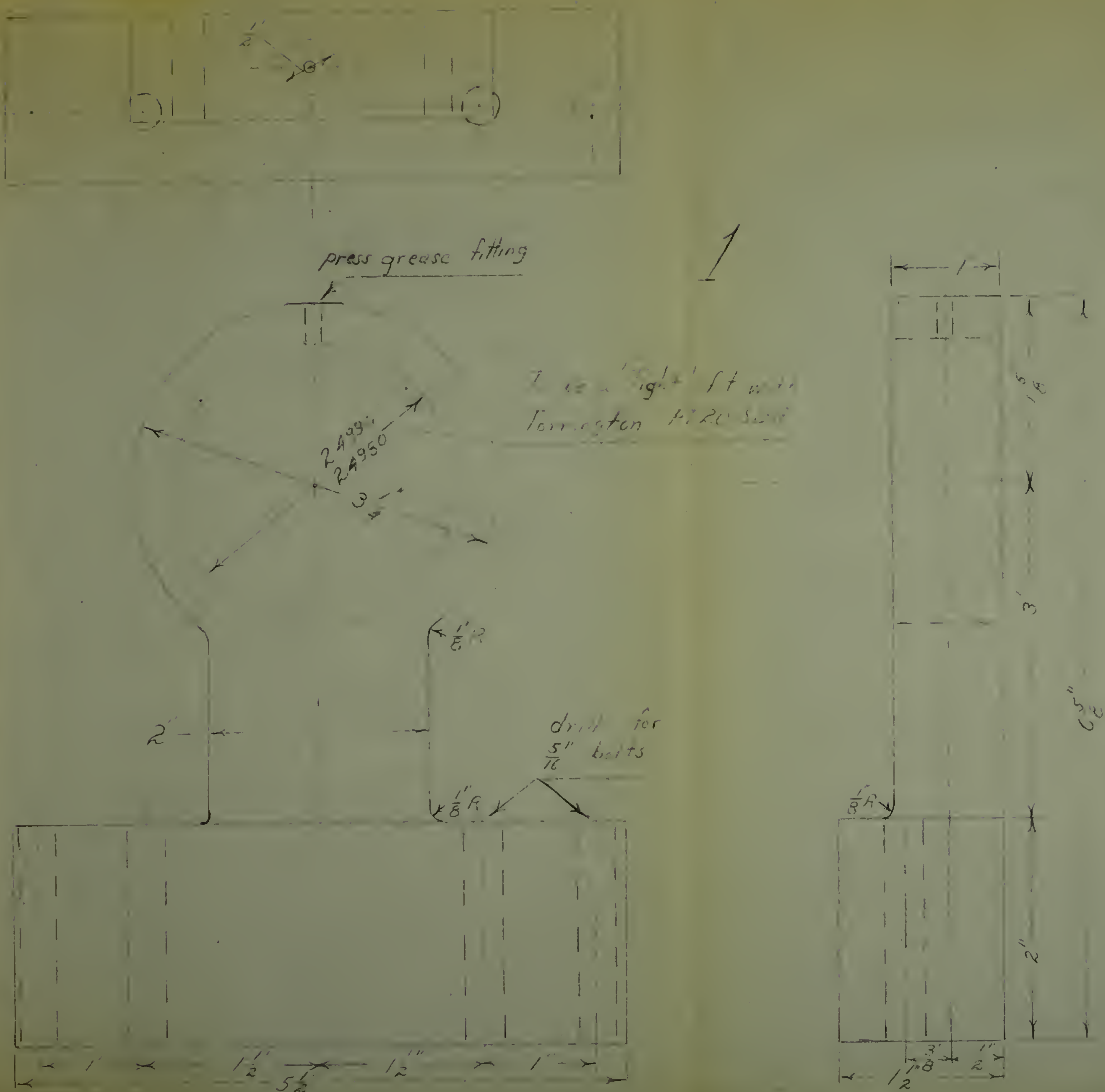
Hathaway Oscillograph Type S-15-B

Baldwin SR-4 Calibration Units

PARTS LIST

Part No.	Name	No. Req'd.	Material	Specification
1	Bearing Holder (fixed)	1	24ST	See Dwg.
2	Bearing Holder (oscillating)	1	24ST	See Dwg.
3	Bearing Support	1	24ST	See Dwg.
4	Bearing Support	1	24ST	See Dwg.
5	Specimen Holder	2	H.T. Steel	See Dwg.
6	Specimen Holder Sleeve	2	H.T. Steel	See Dwg.
7	Specimen Holder Cap	2	H.T. Steel	See Dwg.
8	Torque Pin	4	Drill Rod	See Dwg.
9	Bearing	2		Torrington 20NBK2040YZP
10	1/2 Bolt	2	H.T. Steel	1/2"X20X2" Hex.
11	5/16" Key	4	H.T. Steel	
12	Grease Fitting	8		1/8" Press Fit
13	Torque Arm	2	61ST	See Dwg.
14	Bearing	4		Torrington 12NBK1830YZP
15	Hold Down Pin	4	H.T. Steel	See Dwg.
16	Hold Down Link	4	61ST	See Dwg.
17	Lower Bearing Holder	2	H.T. Steel	See Dwg.
18	Lower Bearing Cap	2	H.T. Steel	See Dwg.
19	Table Clip	4	H.T. Steel	See Dwg.
20	Table Top	2	H.T. Steel	See Dwg.

21	5/16" Bolt	8	H.T. Steel	5/16"X24X2½" Hex
22	7/16" Locknut	2	H.T. Steel	7/16" NF Elastic Stopnut
23	1/4" Machine Screw	8	H.T. Steel	1/4"X20X1/2" Hex.
24	1/4" Bolt	4	H.T. Steel Hex.	1/4"X28X1½"
25	1/4" Bolt	2	H.T. Steel	1/4"X28X2" Hex.
26	1/4" Locknut	6	H.T. Steel	1/4" NF Elastic Stopnut
27	3/16" Machine Screw	3	H.T. Steel	3/16"X30X2½" Hex.
28	3/16" Locknut	3	H.T. Steel	3/16" NF Elastic Stopnut
29	3/8" Bolt	4	H.T. Steel	3/8"X24X2½" Socket Hex.
30	5/8" Bolt	6	H.T. Steel	5/8"X18X1½" Hex.
31	3/4" Bolt	1	H.T. Steel	3/4"X16X1½" Hex.



Notes

Material ~ 24ST-4

All tolerances ± 0.0020 except as specified

Fine machine finish throughout

One (1) required

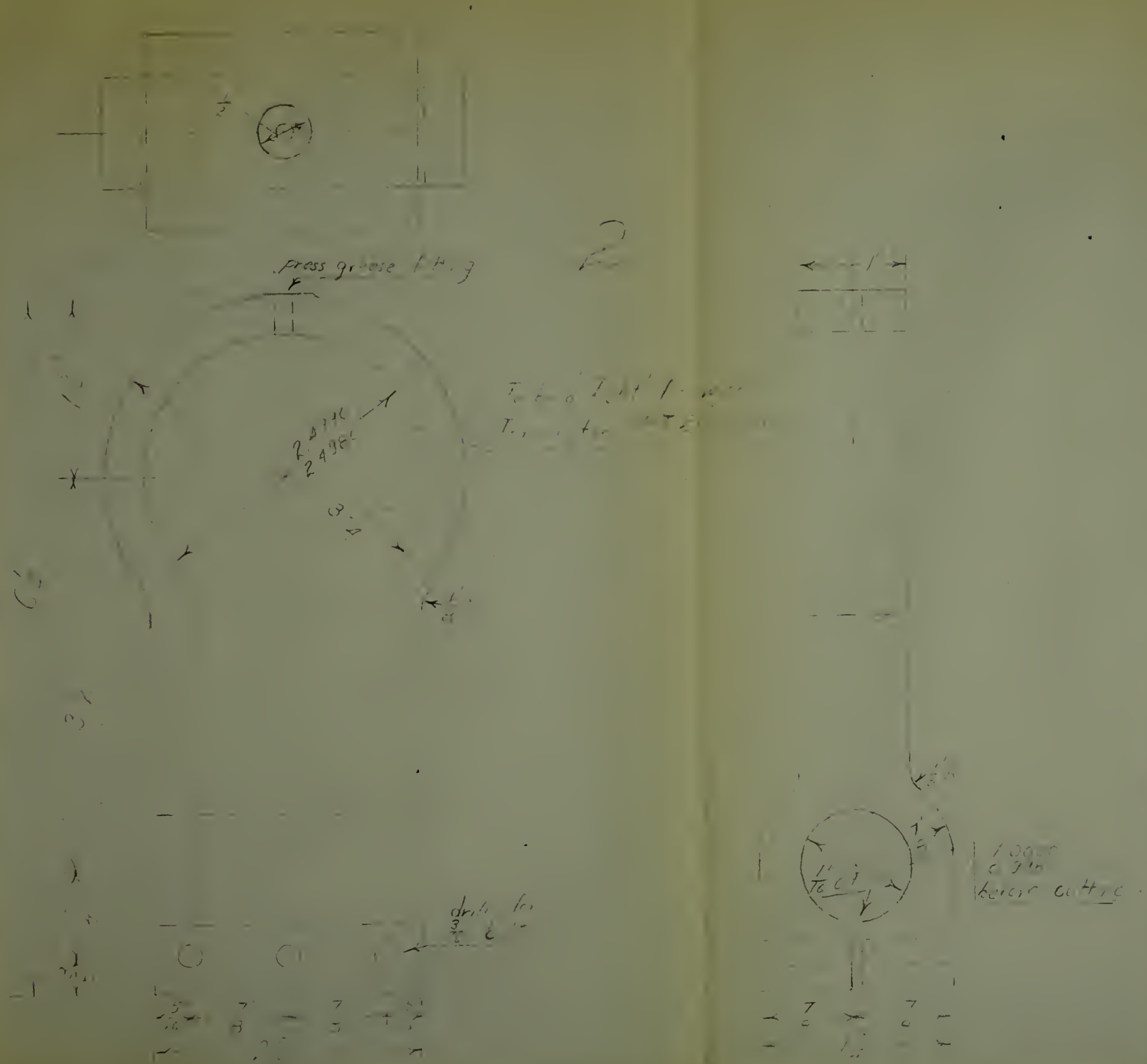
U.S Naval Postgraduate School
Monterey, California

Combined Stress Fatigue Jig

Center bearing housing
(fixed)

A J Gilmore

26 February



Notes:

Material ~ 24ST-4

All tollerances ±0.002 except as specified

Fine machine first [illegible]

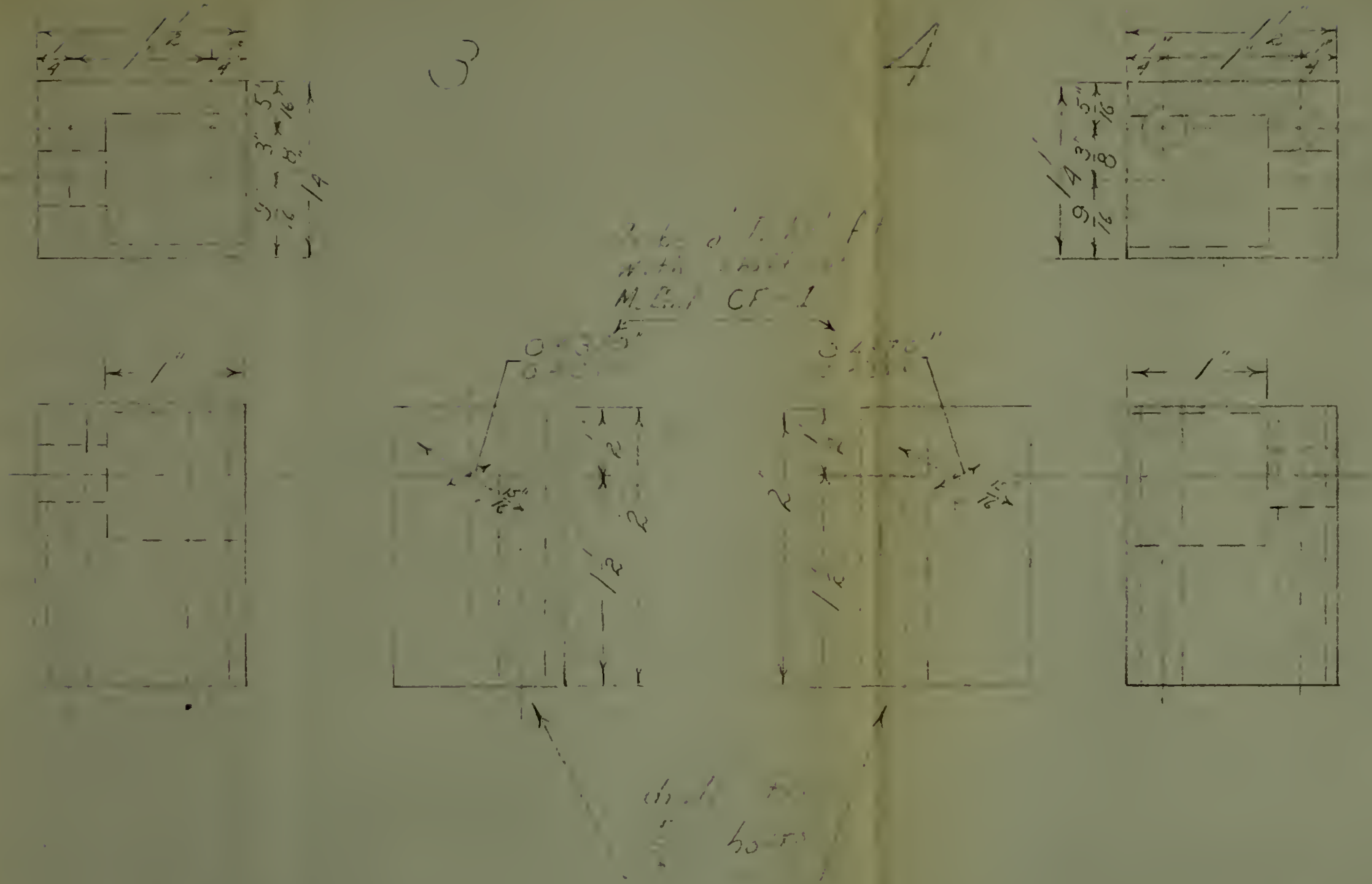
One (1) required

U.S. Naval Postgraduate School
Monterey, California

Combined Stress [illegible] T[illegible]

[illegible] bearing [illegible]

A.J. Gilmore [illegible]



Notes

Material ~ 24ST-4

All tollerances ± 0.0020 except as specified

Fine machine finish throughout

One (1) each required

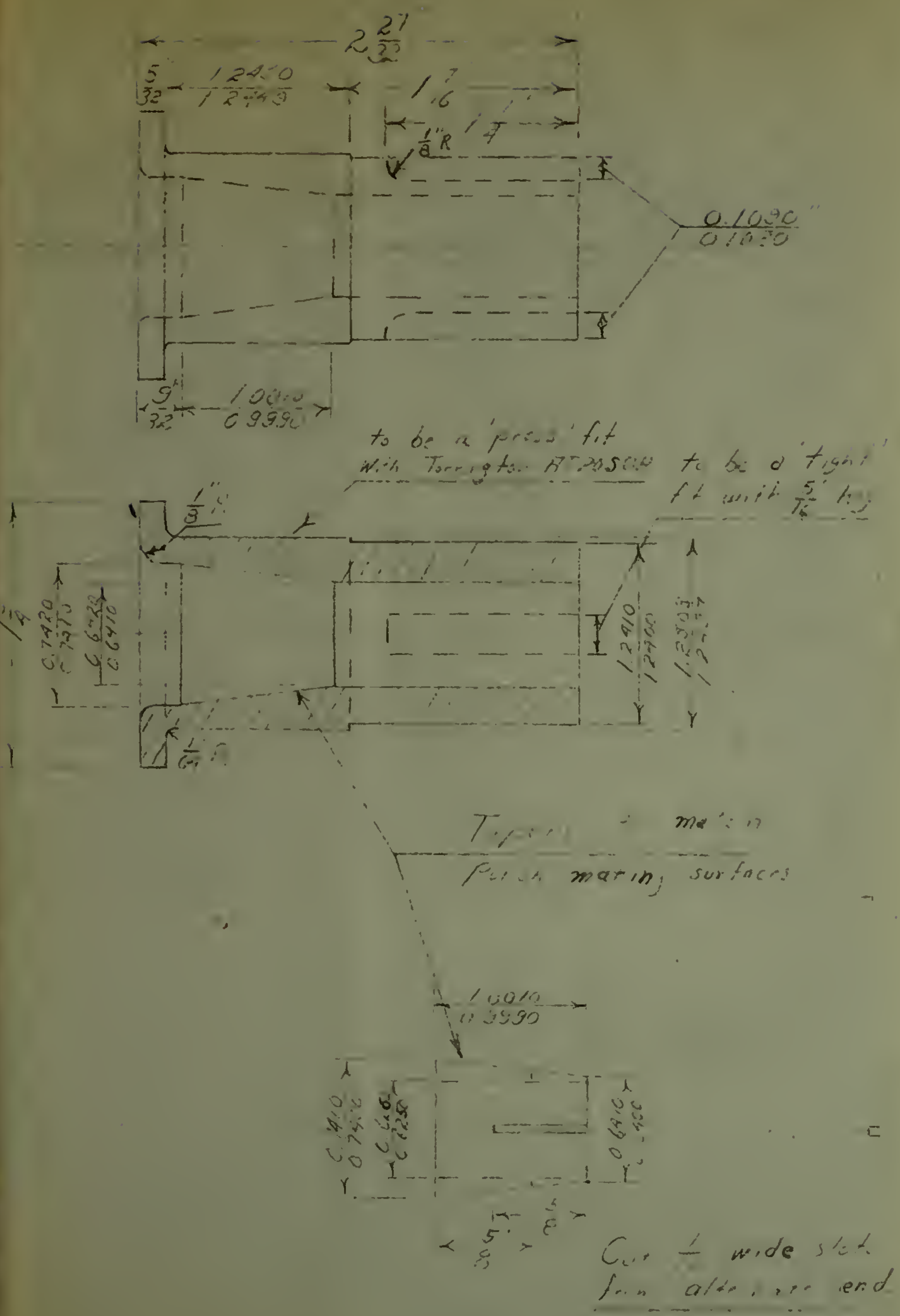
U.S Naval Postgraduate School
Monterey, California

Combined Stress Fatigue Jig

Center bearing supports

R J Gilmore

27 February 19[illegible]



5

6

Notes

Material ~ Cold Rolled Steel

All tollerances ± 0.002 except as specified

Fine machine finish except as specified

Two (2) each required

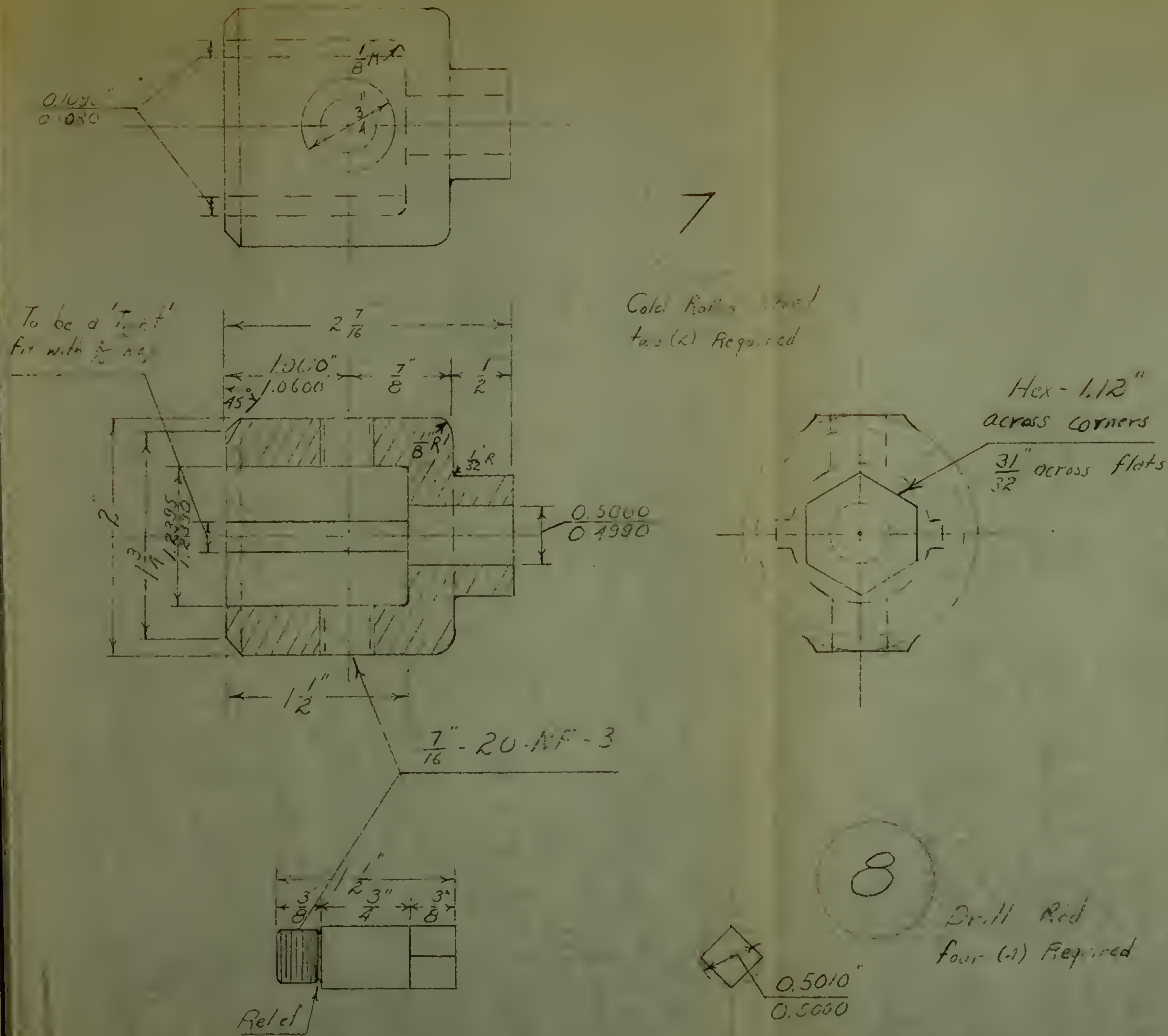
U.S. Naval Postgraduate School
Monterey, California

Combined Stress Fatigue Jig

Specimen holder and sleeve

R.J. Gilmore

27 January 195[illegible]



Notes

Material ~ As indicated

All tollerances ± 0.002 except as specified

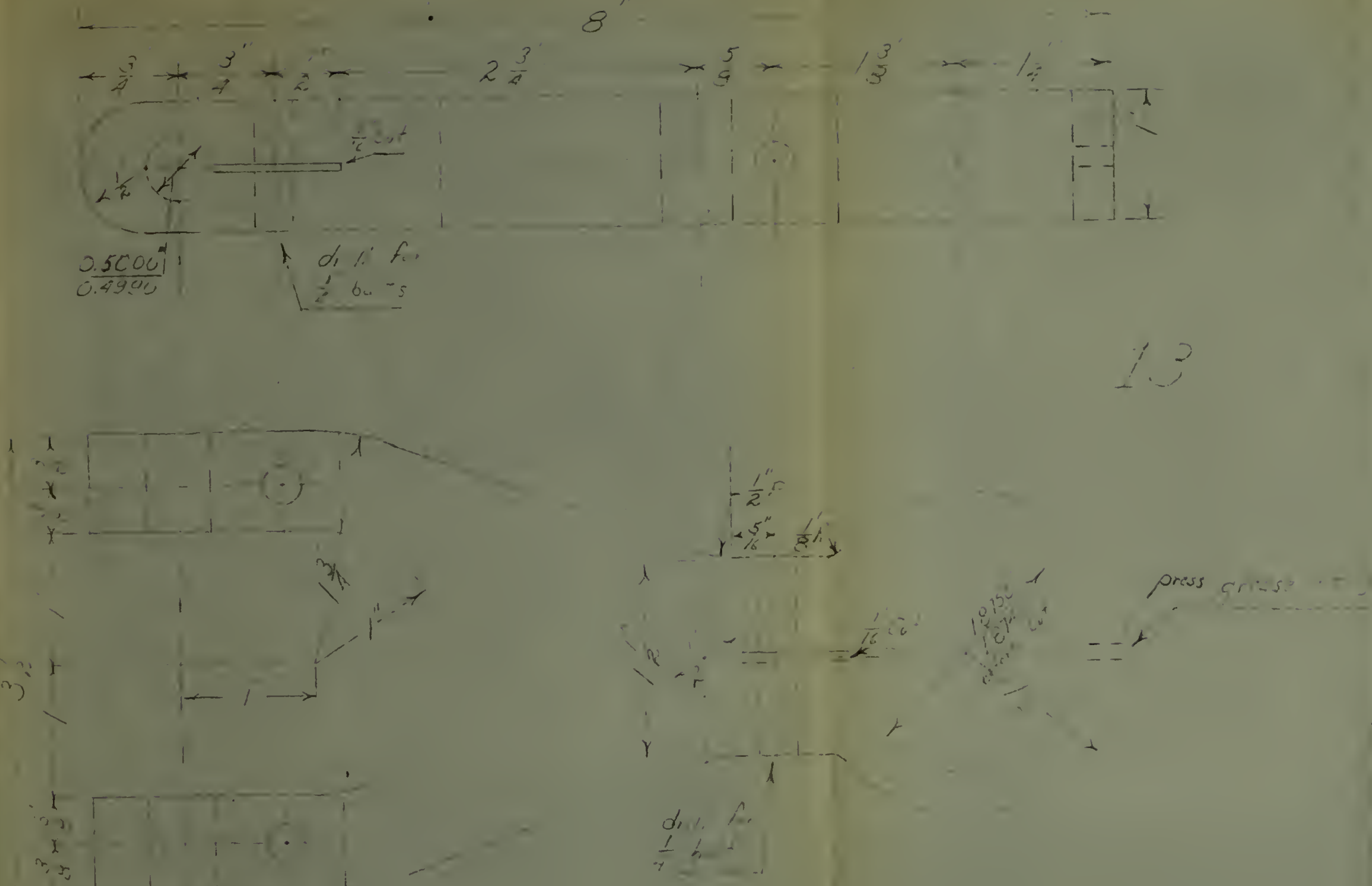
Fine machine finish throughout

U.S. Naval Postgraduate School
Monterey, California

Combined Stress Fatigue Jig

Specimen Holder Cap

A.J. Gilmore — 1 March 1953



Notes

Material ~ 61ST-6

All tollerances ± 0.002 except as specified

Fine machine finish throughout

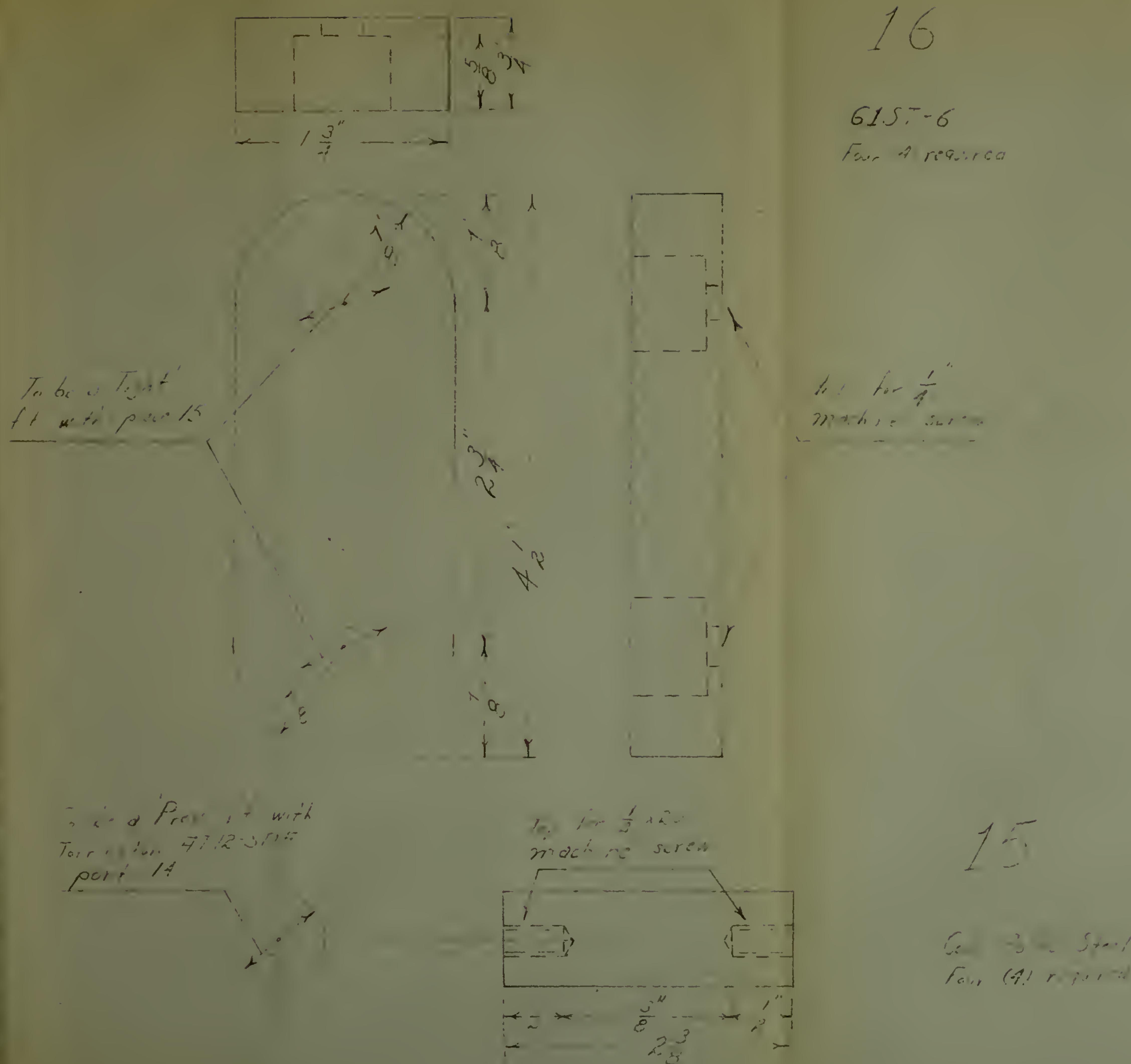
Two (2) required

US Naval Postgraduate School
Monterey California

Combined Stress Fatigue Jig
Torque Arm

A J Gilmore

3 March [illegible]



Material ~ ...

All tollerances ...

Fine machine finish ...

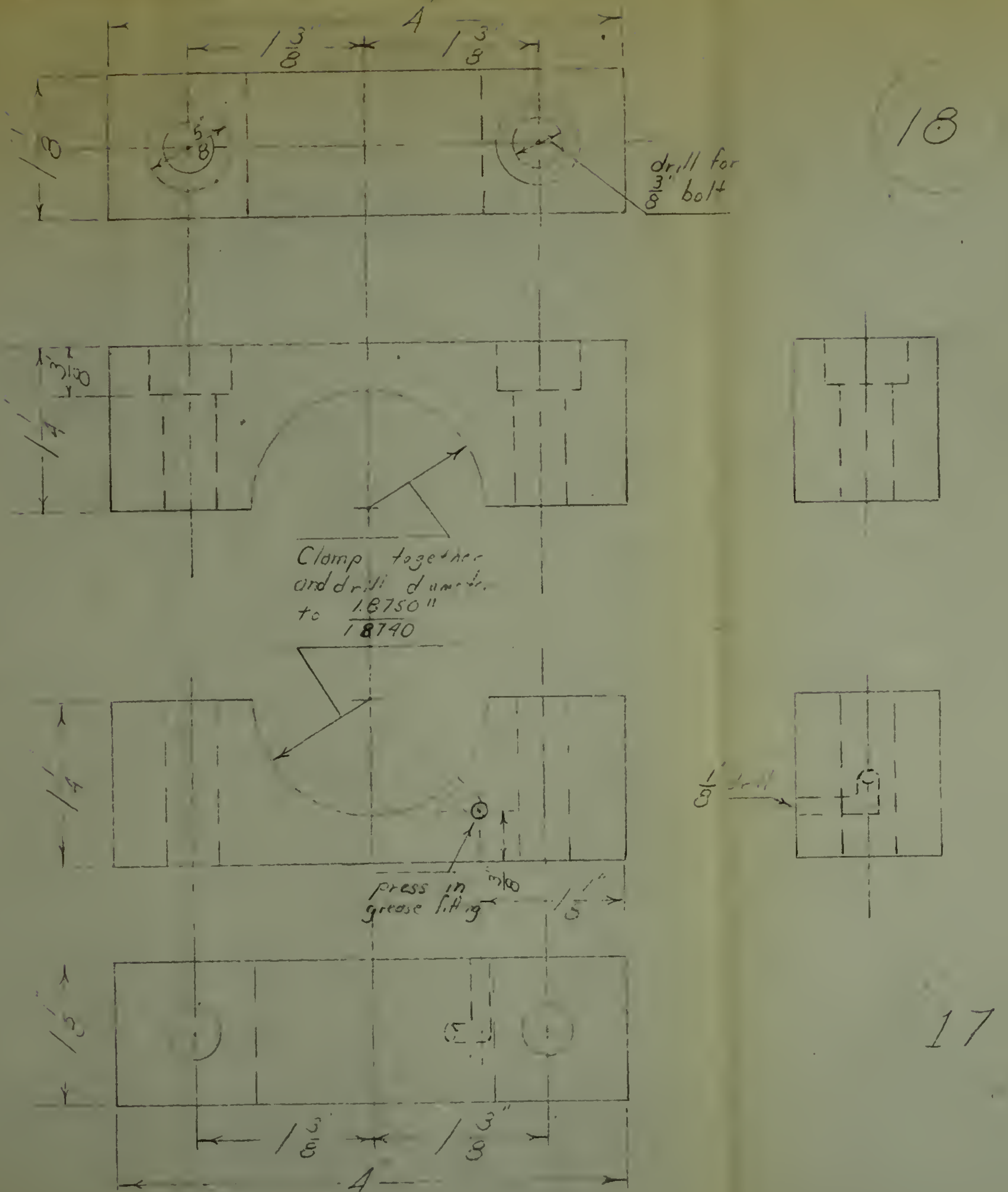
Four (4) each required

U.S Naval Postgraduate School
Monterey, California

Combined Stress Fatigue Jig

A.J Gilmore

3 March 19..



Notes

Material ~ Cold Rolled Steel

All tolerances ± 0.002 except as specified

Fine machine finish throughout

Two (2) each required

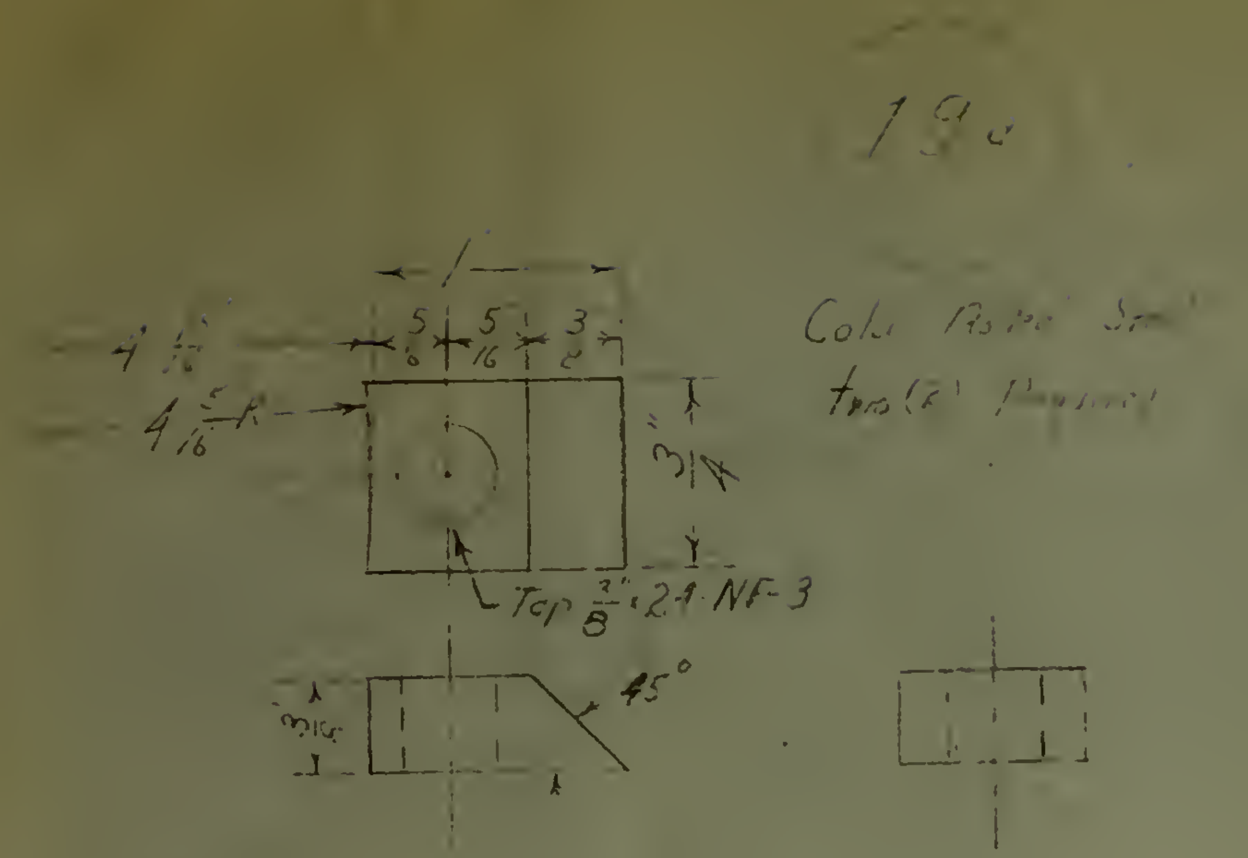
U.S. Naval Postgraduate School
Monterey, California

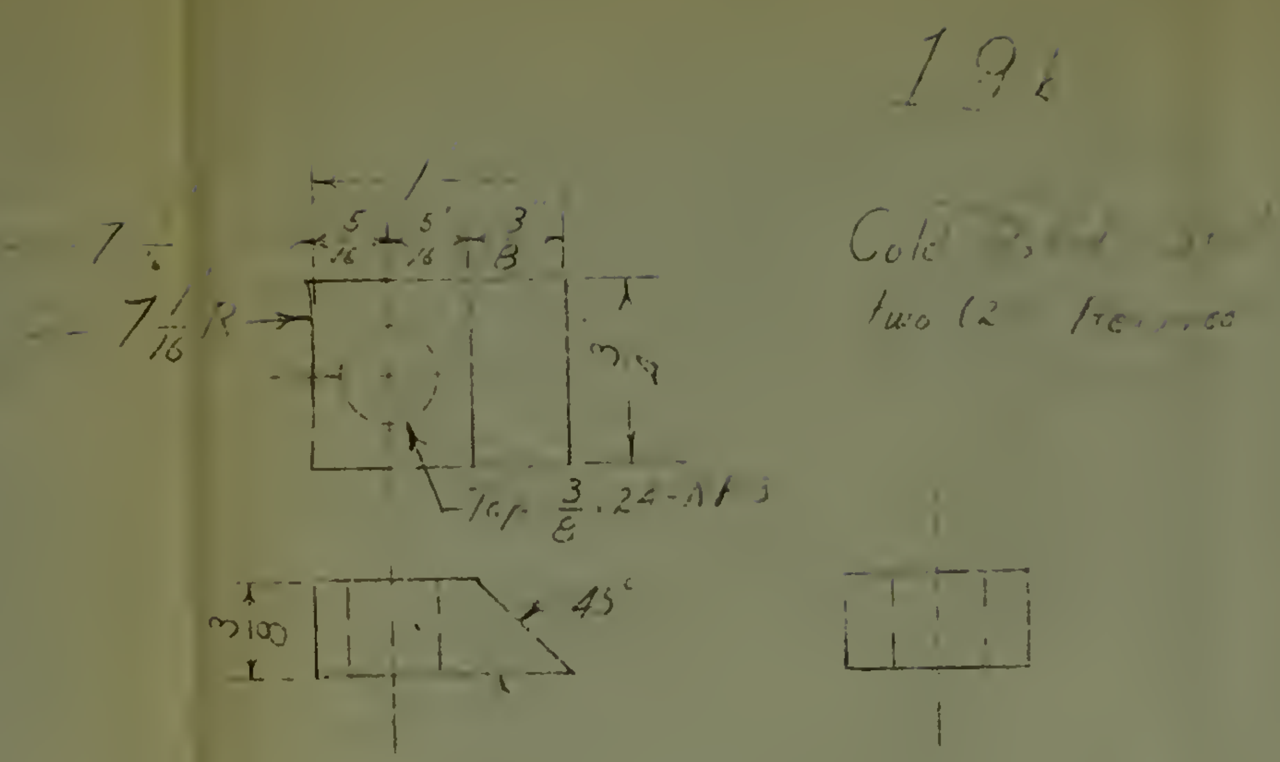
Combined Stress Fatigue Jig

Lower bearing holder and cap

AJ Gilmore

4 March 1953



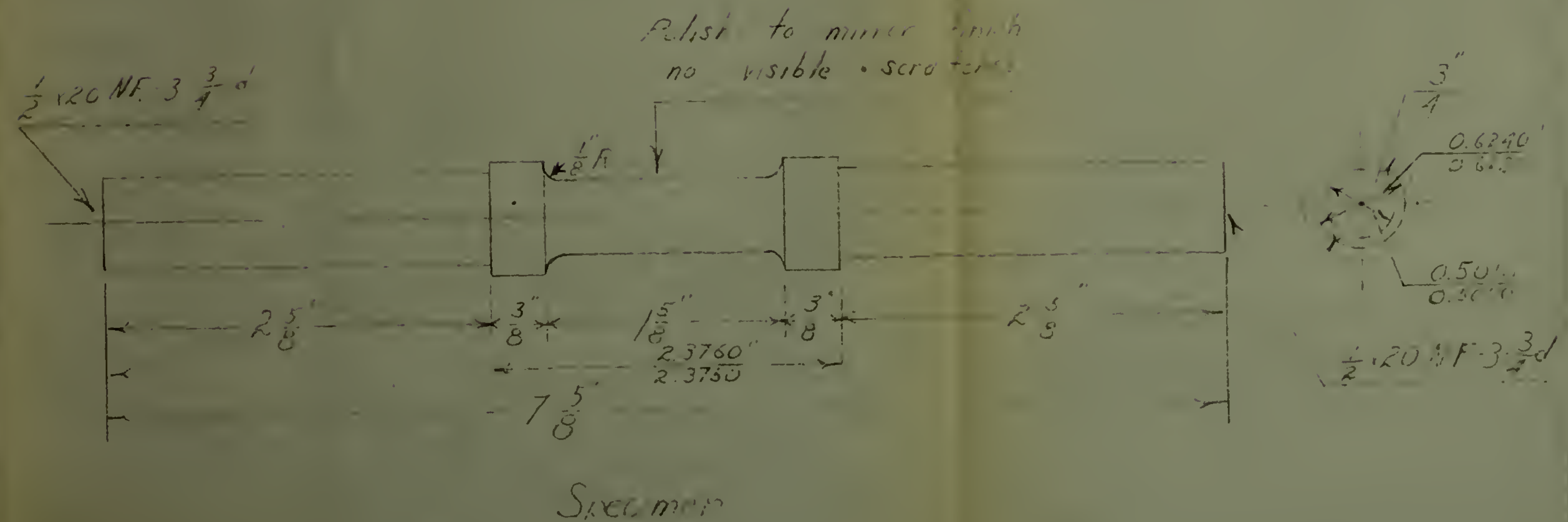


Notes

Material ~ As specified

All tolerances ± 0.0020" except as specified

Fine machine finish throughout

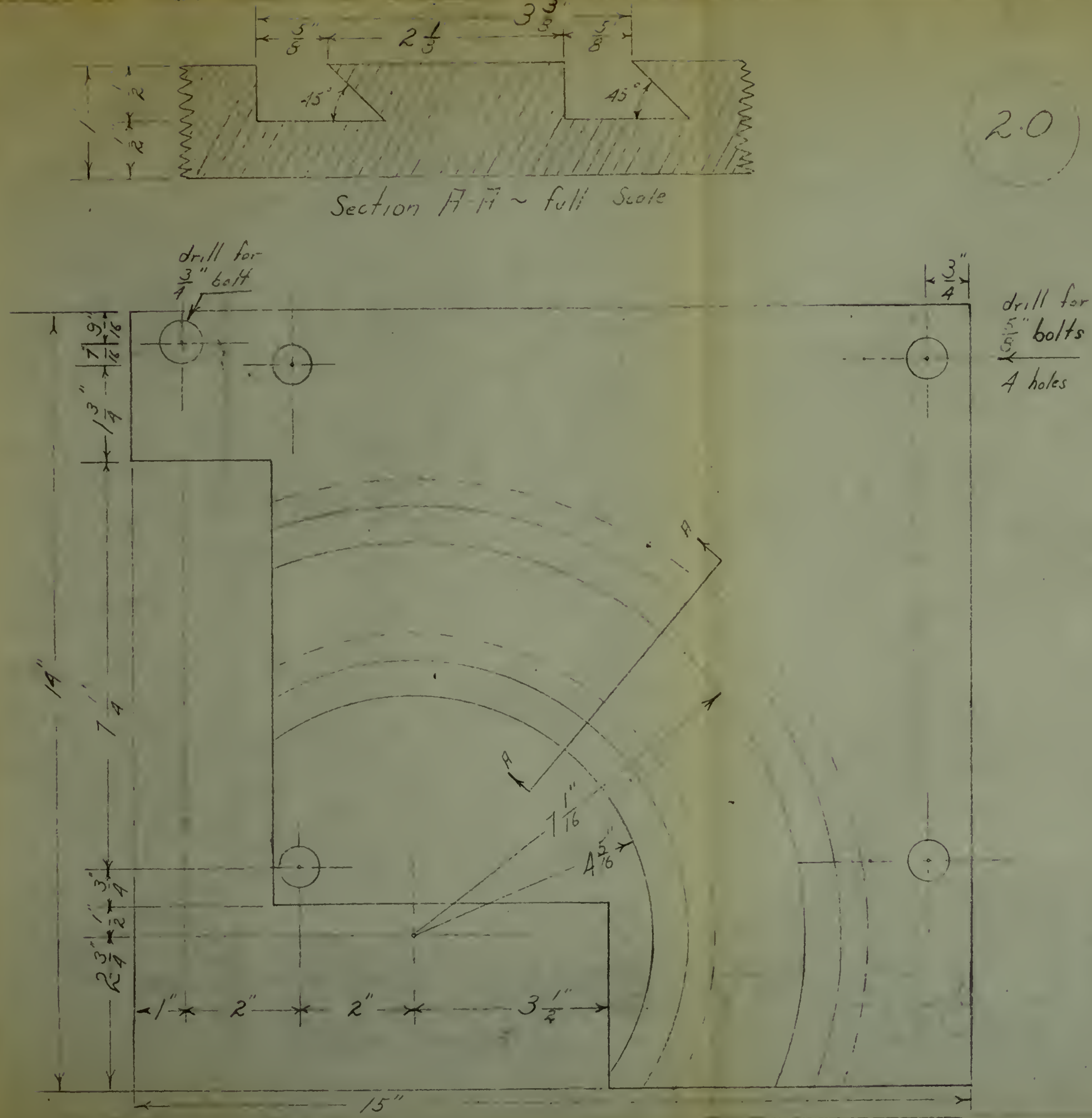


U.S Naval Postgraduate School
Monterey, California

Combined Stress Fatigue Jig

Table clips and Specimen

A.J. Gilmore 5 March 19..



2.0

Notes

Material ~ 1" Steel plate

All tollerances ± 0.0020 except as specified

Fine machine finish throughout

Two (2) required

Half Scale

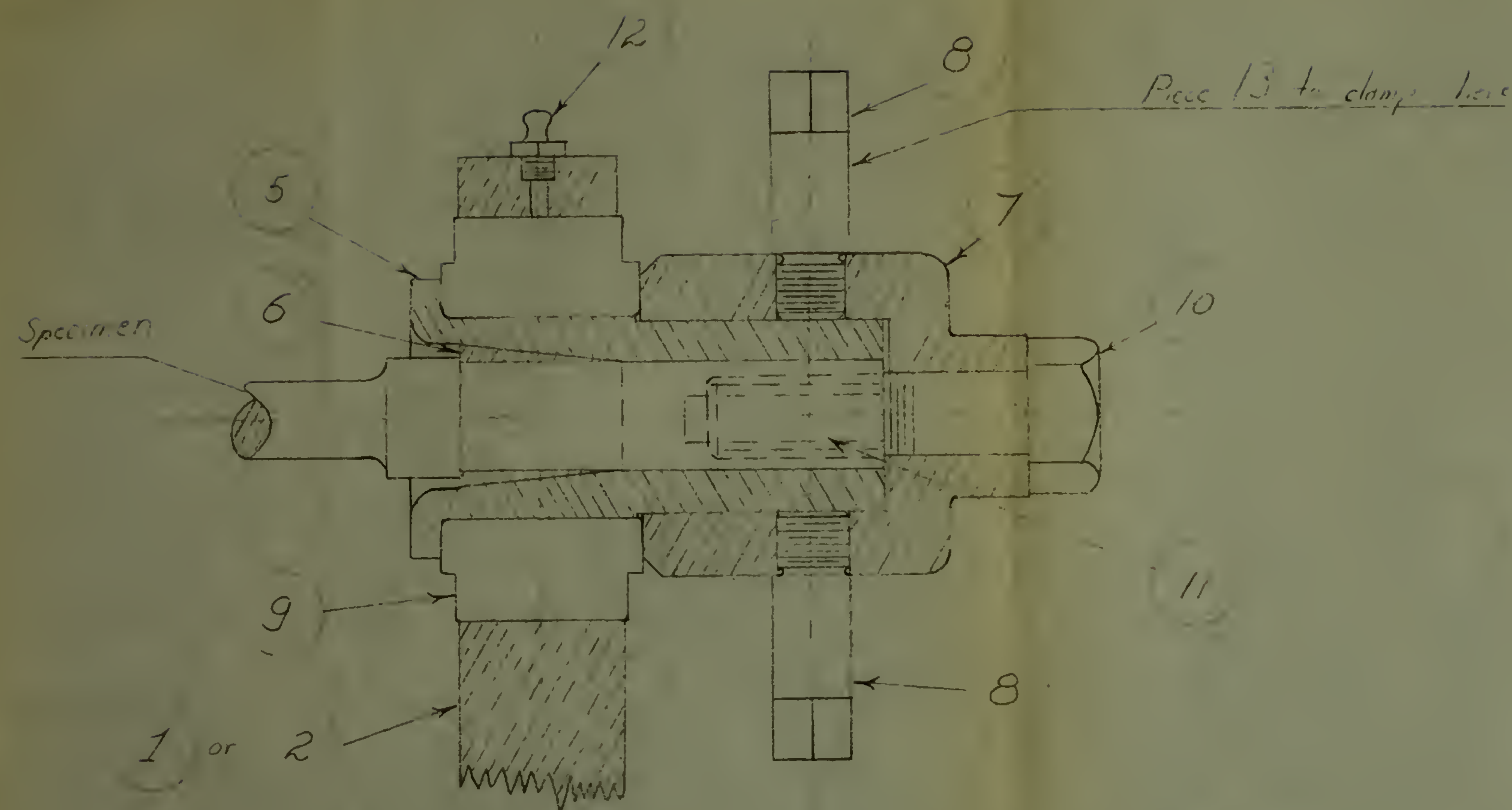
U.S. Naval Postgraduate School
Monterey, California

Combined Stress Fatigue Jig

Table top

A.J. Gilmore

4 March 1953

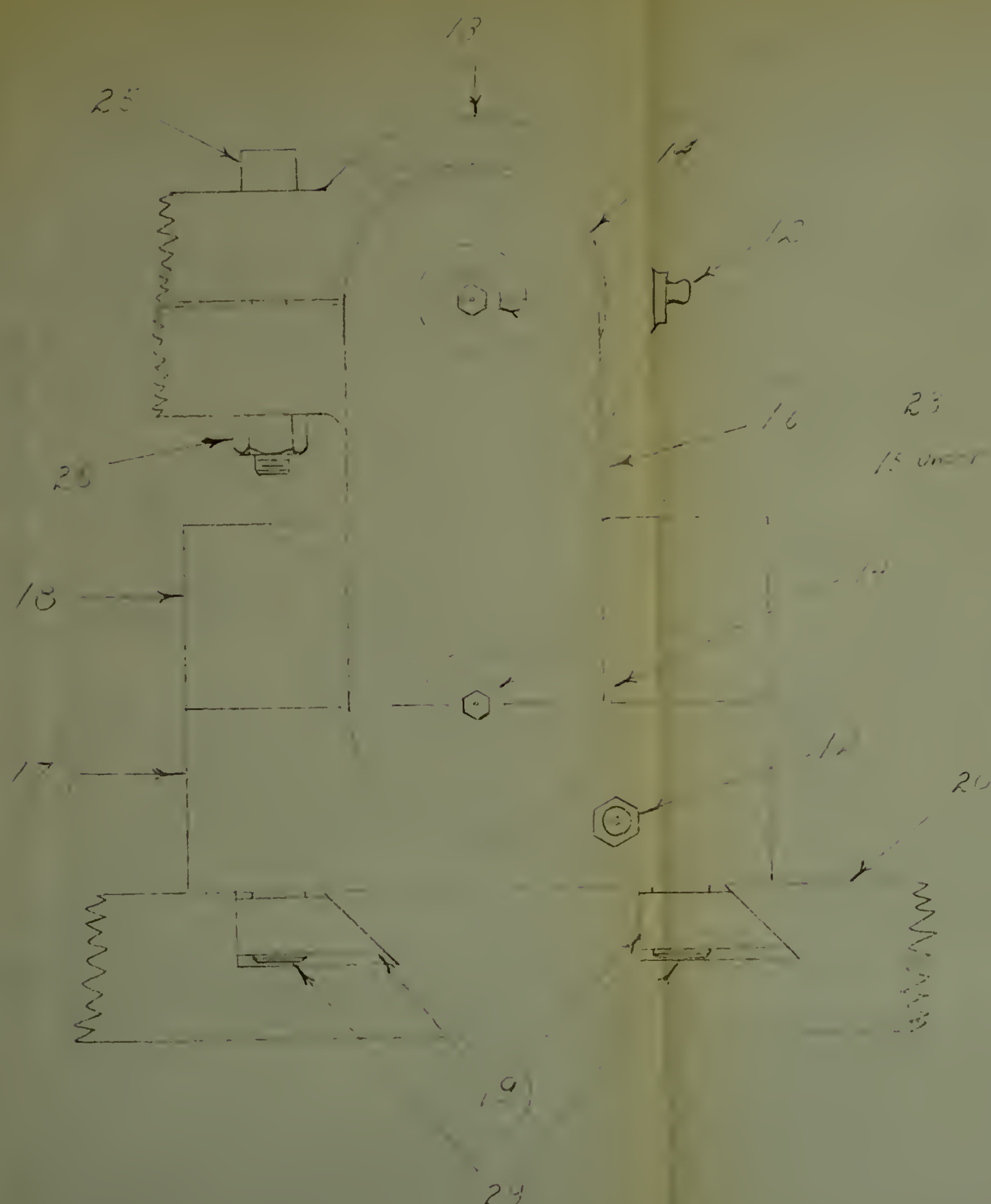


U.S. Naval Postgraduate School
Monterey, California

Combined Stress Fatigue Jig

Assembly of specimen holder

A.J. Gilmore 3 March [illegible]

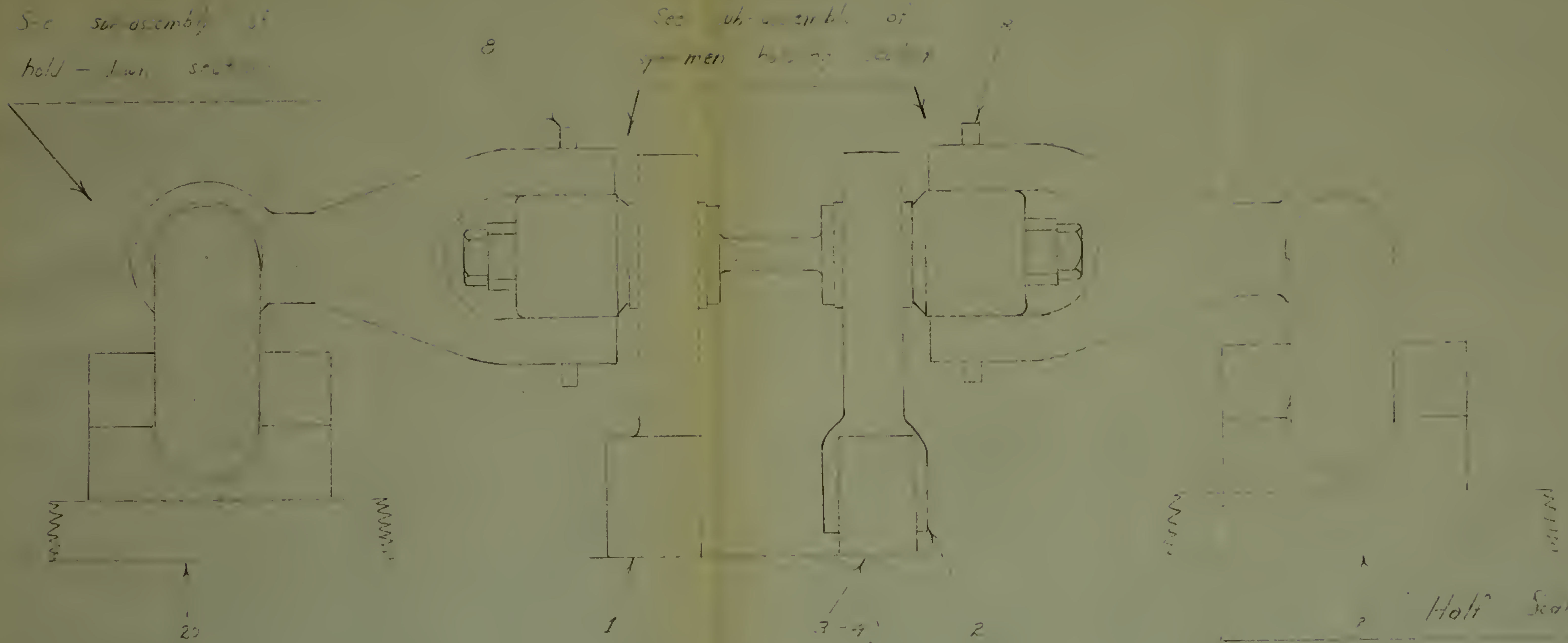


U.S Naval Postgraduate School
Monterey, California

Combined Stress Fatigue Jig

Assembly of hold down

R.J Elmore 5 March [illegible]



Half Scale

U.S. Naval Postgraduate School
Monterey, California

Combined Stress Fatigue Rig

[illegible]

R. J. Elmore [illegible]